J. KSIAM Vol. 13 No. 2, June. 2009, pp.73~86

THERMAL INSTABILITY IN REACTIVE VISCOUS PLANE POISEUILLE / COUETTE FLOWS FOR TWO EXTREME THERMAL BOUNDARY CONDITIONS

SURAJU OLUSEGUN AJADI

The Korean Society for Industrial and Applied Mathematics

J. KSIAM Vol.13, No.2, 73–86, 2009

THERMAL INSTABILITY IN REACTIVE VISCOUS PLANE POISEUILLE / COUETTE FLOWS FOR TWO EXTREME THERMAL BOUNDARY CONDITIONS

SURAJU OLUSEGUN AJADI

DEPARTMENT OF MATHEMATICS, OBAFEMI AWOLOWO UNIVERSITY, NIGERIA
E-mail address: sajadi@oauife.edu.ng, soajadi@yahoo.co.uk

ABSTRACT. The problem of thermal stability of an exothermic reactive viscous fluid between two parallel walls in the plane Poiseuille and Couette flow configurations is investigated for different thermal boundary conditions. Neglecting reactant consumption, the closed-form solutions obtained from the momentum equation was inserted into the energy equation due to dissipative effect of viscosity. The resulting energy equation was analyzed for criticality using the variational method technique. The problem is characterized by two parameters: the Nusselt number(N) and the dynamic parameter(Λ). We observed that the thermal and dynamical boundary conditions of the wall have led to a significant departure from known results. The influence of the variable pre-exponential factor, due to the numerical exponent m, also give further insight into the behavior of the system and the results expressed graphically and in tabular forms.

1. INTRODUCTION

The fully developed two-dimensional plane reactive viscous flow between parallel walls is still receiving attentions because of its significance to kinematics, the variability of thermodynamics and transport properties, thereby making it vital and fundamental to all engineering and applied scientific studies(Laminar combustion, furnaces, lubricants hydrodynamics e.t.c). In particular, safe storage and transportation of combustible and potentially explosive materials (such as fuel), the efficient and reliable operation of practical devices, all require an improved understanding of the combustion process. Generally speaking, most lubricants used in engineering and industrial processes are reactive(hydrocarbon oils, synthetic esters e.t.c), and their efficiency depends on temperature variations due to thermal and dynamical conditions of the enclosing walls[Makinde-12]. This study can also find its place in the area of pneumatic conveyor system, which makes use of pipes or ducts called transportation lines that carry mixture of materials and a stream of air.

Received by the editors February 12 2009; Accepted April 25 2009.
2000 *Mathematics Subject Classification.* 80A25, 80A20.
Key words and phrases. Thermal ignition, Poiseuille/Couette flow, Variational method, Viscous dissipation, Pre-exponential factor.
The author was supported in part by the Skirball postdoctoral fellowship of the Center of Advanced Studies in Mathematics, Ben-Gurion University.

The developed plane flow between two flat parallel surfaces is well understood for the case where one surface is moving parallel to the other (Couette flow); such that flow is driven by differential tangential motion of enclosing walls[Fang-6]. In a similar way the Poiseuille flow is often associated with rectilinear pressure driven flows in stationary conduits[Rodia and Osterle-17].

In the study of thermal ignition (criticality), there are specific criteria which determine spontaneous change in behavior [1, 2, 12, 15, 16]. Thermal ignition is a type of instability in which the combustible at a negligible rate is brought to a condition in which it is reacting at an appreciable rate. In the study of thermal ignition, the neglect of the momentum of the reacting species is well established. This is because of the assumption that no momentum is created by the chemical reactions[9]. However, in a reactive viscous flow, it may be very significant to include the momentum equation of the reactive system to accommodate the flow dynamics. Since the enclosing wall(s) may be stationary (Poiseuille flow) or subjected to uniform motion (Couette flow), it would be challenging to investigate the impact of these variations on the thermal ignition of the system.

Historically, the theory of combustion was developed through studies of simpler models with additional and often unfounded assumptions. However, in this work, the assumption of constant density approximation characterizes combustion as a low speed phenomenon(small Mach number) in a chemically reacting mixture which is valid for situations where the hydrodynamical effects play a secondary role with respect to the reactive and diffusive effect[5, 18]. It is often employed as a substantially simplifying assumption to decouple the momentum equation from the energy equation[12, 13]. In addition, the assumption of low Mach number provided that the heat release is sufficiently weak and with this, the hydrodynamic flow field decouples from the heat equation to leading order.

Okoya and Ajadi[16] examined the thermal stability of two models, whose thermal conductivity is a function of temperature. They showed the way in which thermal explosion is affected by boundary conditions and other parameters. Furthermore, Okoya[14] considered the thermal stability for a reactive viscous, Newtonian flow in a slab for a plane Poiseuille flow. It was observed that criticality parameter and the excess maximum temperature are monotonically increasing function of the non-Newtonian coefficient.

More recently, Makinde[11] studied the steady state solutions for a strongly exothermic viscous reactive flows through channels with sliding wall. They revealed accurately, the steady state thermal criticality conditions for viscous reactive effectiveness using the perturbation summation and improvement technique.

Motivated by the above, we considered thermal stability behaviors of an exothermic combustible material between parallel channels based on the Couette and Poiseuille flows configurations. Parametric analysis was conducted using the Nusselt number(N), the dynamic parameter(Λ) and the numerical index m(due to the temperature dependent pre-exponential factor), on the thermal ignition behavior of the system and the results expressed graphically and in tabular forms.

2. Governing Equations

We consider a reactive system based on a one-step reaction mechanism of the form,

$$F + O \xrightarrow{w_i} P,\ w_i = A(T)[F][O], \tag{2.1}$$

enclosed between two parallel plates, which are hydrodynamically and thermally developed uni-directional flow in the x-direction. In the reaction (2.1), F and $[F]$ represent the fuel and fuel concentrations respectively, O and $[O]$ represent the oxidant and oxidant concentrations respectively, w_i is the reaction rates, $A(T)$ is the variable pre-exponential factor.

Assuming negligible reactant consumption, the steady state governing continuity, momentum and energy equations are:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{2.2}$$

$$\rho\left(u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right) = -\frac{dP}{dx} + \mu\frac{\partial^2 u}{\partial y^2}, \tag{2.3}$$

and

$$\frac{d^2T}{dy^2} + \frac{QC_0A(T)}{K}\exp\left(\frac{-E}{RT}\right) + \frac{\mu}{K}\left(\frac{du}{dy}\right)^2 = 0. \tag{2.4}$$

The appropriate boundary conditions for the momentum and energy equations are,

$$u(0) = 0,\ u(a) = 0, \tag{2.5}$$

$$u(0) = 0,\ u(a) = V_p, \tag{2.6}$$

$$\frac{dT}{dy} + \frac{H}{K}(T - T_0) = 0,\ y = 0,\ \text{and } T = T_0,\ y = a, \tag{2.7}$$

where T is the absolute temperature variable, $\frac{dP}{dx}$ is the pressure gradient(decreasing), T_0 is the wall temperature, K is the thermal conductivity of the combustible material, $Q(T)$ is the heat release, A is the pre-exponential factor, E is the activation energy, R is the universal gas constant, a is the distance between the plates and C_0 is the initial concentration of the reactant, H is the heat transfer coefficient, μ is the combustible material dynamic viscosity coefficients, V_p is the velocity of the upper plate, u and v are the velocity along and across the channel respectively, ρ is the constant density and the ratio $\frac{H}{K}$ is the Nusselt number(N). N is a dimensionless version of the temperature gradient at the surface between the fluid and the solid, and it thus provides a measure of the convection occurring from the surface. Equations (2.5) and (2.6) represent the Poiseuille and Couette flows respectively. Condition (2.7) at $y = 0$ is the general Newtonian exchange of heat at the wall surface, while condition (2.7) at $y = a$ corresponds to the perfect heat transfer on the boundary. In the first part of (2.7), the two extreme values for the Nusselt number N have been considered: $N = 0$ corresponds to adiabatic heat transfer($H = 0$), while $N = \infty$ refers to the perfect heat transfer($K = 0$) on the boundary[2, 7, 16].

Assuming that the velocity across the channel is space invariant($\frac{dv}{dy}$=0), we can non-dimensionalize equations (2.2) - (2.6) using the variables,

$$\theta = \frac{E(T-T_0)}{RT_0^2},\ y' = \frac{y}{a},\ x' = \frac{x}{a},\ W = \frac{u}{V_p}. \tag{2.8}$$

By dropping primes, the dimensionless continuity, momentum and energy equations become

$$\frac{\partial W}{\partial x} = 0, \Rightarrow W = W(y), \tag{2.9}$$

$$\frac{d^2W}{dy^2} - \alpha\frac{dW}{dy} + G = 0, \tag{2.10}$$

and

$$\frac{d^2\theta}{dy^2} + \delta\left[(1+\epsilon\theta)^m \exp\left(\frac{\theta}{1+\epsilon\theta}\right) + \beta\left(\frac{dW}{dy}\right)^2\right] = 0. \tag{2.11}$$

The appropriate boundary conditions are as follows:

$$W(0) = 0,\ W(1) = \Lambda = 0 \implies \text{Poiseuille flow.} \tag{2.12}$$

$$W(0) = 0,\ W(1) = \Lambda = 1 \implies \text{Couette flow.} \tag{2.13}$$

$$\left.\frac{\partial\theta}{\partial y}\right|_{y=0} = 0,\ \theta(1) = 0, \implies N = 0, \tag{2.14}$$

$$\theta(0) = 0,\ \theta(1) = 0 \implies N \to \infty, \tag{2.15}$$

where ϵ(dimensionless activation energy), α(heat convection), $\Lambda = \frac{u(a)}{V_p}$(plate motion parameter), β(viscous dissipation), δ(Frank-Kamenetskii parameter) are given by

$$\epsilon = \frac{RT_0}{E}, \alpha = \frac{\rho a V_p}{\mu},\ G = \frac{a^2}{\mu V_p}\frac{dP}{dx},\ \beta = \frac{\mu V_p^2}{QC_0A_0a^2},\ \text{and } \delta = \frac{QEC_0A_0T_0^{m-2}a^2}{KR\exp(\frac{E}{RT_0})}.$$

3. Methods of Solutions

3.1. **Momentum Equations(closed-form).** The momentum equation (2.10) and boundary conditions (2.12) and (2.13), after integration, lead to closed form solutions of the forms;

$$W(y) = \left(\Lambda + \frac{G}{2}\right)y - \frac{G}{2}y^2,\ \alpha = 0 \tag{3.1}$$

$$W(y) = G\frac{y}{\alpha} + \frac{\Lambda\alpha - G}{\alpha(e^\alpha - 1)}(e^{\alpha y} - 1),\ \alpha \neq 0, \tag{3.2}$$

where $\Lambda = 0$ and $\Lambda = 1$ correspond to the Poiseuille and Couette flow respectively. The graphical expressions of the solutions (3.1) and (3.2), shown in Figures 9 and 10 respectively.

3.2. **Energy Equations(Variational method).** In a simplified system, the energy equation for a one-step reaction system is often amenable to closed-form solution. However, the inclusion of the momentum equation in this consideration makes this unrealistic. We are motivated to use the variational technique[1, 2, 13], which is based on the maximum principle and often applicable to problems of these types. Thus, the mathematical criteria for maximum critical ignition points is

$$\frac{d\delta}{d\theta} = 0, \ \frac{d^2\delta}{d\theta^2} < 0, \tag{3.3}$$

from which the calculation of maximum temperature (θ_{cr}) and the corresponding critical parameter (δ_{cr}) are obtained. And beyond θ_{cr} and δ_{cr}, we we have the onset of thermal instability. Before transforming the problem to a variational form, for simplicity, we let

$$f(\theta) = (1+\epsilon\theta)^m \exp\left(\frac{\theta}{1+\epsilon\theta}\right) + \beta\left(\frac{dW}{dy}\right)^2, \tag{3.4}$$

$$F(\theta) = \int f(\theta)d\theta = \int \left[(1+\epsilon\theta)^m \exp\left(\frac{\theta}{1+\epsilon\theta}\right) + \beta\left(\frac{dW}{dy}\right)^2\right] d\theta.$$

Consider the Hamiltonian for (2.11)-(2.15) to be

$$H_\delta(\theta) = \int_D \left(\frac{1}{2}|\nabla\theta|^2 - \delta F(\theta)\right) dV, \tag{3.5}$$

and substituting $\theta = \sum A_k\theta_k$ such that $H_\delta = H_\delta(A_1, A_2, \cdots A_n)$ and $A_1, A_2, \cdots A_n$ are the solutions of the system. The variational principle suggests that A_1, A_2,...A_n be determined as the solution of the system

$$\frac{\partial H_\delta}{\partial A_k} = 0, \ k = 1, 2, ..n, \tag{3.6}$$

giving an approximate solution $\theta = \sum A_k\theta_k$ corresponding to the chosen δ and the more the number of A_k, the better the approximation. In reality and for simplicity, the number of A_k are often two(A_1, B_1 and A_2, B_2). The condition determining criticality is according to the implicit function theorem is

$$\frac{\partial^2 H_\delta}{\partial A_k \partial A_l} = 0, \ k, \ l = 1, 2, .., n. \tag{3.7}$$

The domain of H_δ is restricted to those functions satisfying the boundary conditions (2.14); an example of such a function is

$$\theta(\rho) = A_1 \cos(\frac{\pi y}{2}) + B_1 \cos(\frac{3\pi y}{2}). \tag{3.8}$$

Hence,

$$H_\delta(A_1, B_1) = \frac{1}{8}\pi^2(uA_1^2 + 6A_1B_1v + 9B_1^2w) - \delta\int_0^1 \rho^j G(\theta)d\rho, \tag{3.9}$$

where

$$u=\int_0^1 \rho^j \sin^2\left(\frac{\pi\rho}{2}\right)d\rho=\begin{cases}\frac{1}{2},\ j=0,\\ \frac{1}{2(j+1)}+\frac{1}{\pi^2}\ j=1,2,\end{cases}$$

$$v=\int_0^1 \rho^j \sin\left(\frac{\pi\rho}{2}\right)\sin\left(\frac{3\pi\rho}{2}\right)d\rho=\begin{cases}0,\ j=0,\\ \frac{-1}{\pi^2},\ j=1,\\ \frac{-5}{4},\ j=2,\end{cases} \tag{3.10}$$

$$w=\int_0^1 \rho^j \sin^2\left(\frac{3\pi\rho}{2}\right)d\rho=\begin{cases}\frac{1}{2},\ j=0,\\ \frac{1}{2(j+1)}+\frac{1}{9\pi^2},\ j=1,2.\end{cases}$$

The simultaneous equations to be solved for A_k, B_k ($A_k+B_k=\theta_{cr}$) and δ_{cr}, are;

$$\frac{\partial H_\delta}{\partial A_k}=0,\ \frac{\partial H_\delta}{\partial B_k}=0, \tag{3.11}$$

and

$$\left(\frac{\partial^2 H_\delta}{\partial A_k^2}\right)\left(\frac{\partial^2 H_\delta}{\partial B_k^2}\right)=\left(\frac{\partial^2 H_\delta}{\partial A_k\partial B_k}\right). \tag{3.12}$$

Substituting (3.8) - (3.10) into equations (3.11) - (3.12), A_1, B_1 (A_1+B_1=θ_{cr}) and δ_{cr} may be determined from the numerical solutions of the simultaneous equations,

$$\frac{1}{4}\pi^2(uA_k+3vB_k)-\delta\frac{\partial}{\partial A_k}\left(\int_0^1 F(\theta)dy\right)=0, \tag{3.13}$$

$$\frac{3}{4}\pi^2(vA_k+3wB_k)-\delta\frac{\partial}{\partial B_k}\left(\int_0^1 F(\theta))dy\right)=0, \tag{3.14}$$

$$\begin{aligned}&\left(\tfrac{\pi^2 u}{4}-\delta\tfrac{\partial^2}{\partial A_k^2}\int_0^1 F(\theta)dy\right)\times\left(\tfrac{9\pi^2 w}{4}-\delta\tfrac{\partial^2}{\partial B_k^2}\int_0^1 F(\theta)dy\right)\\ &=\left(\tfrac{3\pi^2 v}{4}-\delta\tfrac{\partial^2}{\partial A_k\partial B_k}\int_0^1 F(\theta)dy\right)^2.\end{aligned} \tag{3.15}$$

Similarly, the boundary equation (2.4) is satisfied by,

$$\theta(\rho)=A_2\sin(\pi\rho)+B_2\sin(2\pi\rho), \tag{3.16}$$

where the constants A_2, B_2 (A_2+B_2=θ_{cr}) and δ_{cr} are constants to be calculated. Inserting equation (3.16) into (3.5), we obtain

$$H_\delta(A_2,B_2)=\pi^2(uA_2^2+2vA_2B_2+4wB_2^2)-\delta\int_0^1 \rho^j F(\theta)d\rho, \tag{3.17}$$

$$u = \int_0^1 \rho^j \cos^2(\pi\rho)d\rho = \begin{cases} \frac{1}{2},\ j = 0, \\ \frac{1}{4},\ j = 1, \\ \frac{1}{6} + \frac{1}{4\pi^2},\ j = 2, \end{cases}$$

$$v = \int_0^1 \rho^j \cos(\pi\rho)\cos(2\pi\rho)d\rho = \begin{cases} 0,\ j = 0, \\ \frac{-10}{9\pi^2},\ j = 1, 2, \end{cases} \tag{3.18}$$

$$w = \int_0^1 \rho^j \cos^2(2\pi\rho)d\rho = \begin{cases} \frac{1}{2},\ j = 0, \\ \frac{1}{4},\ j = 1, \\ \frac{1}{6} + \frac{1}{16\pi^2},\ j = 2. \end{cases}$$

Substituting (3.16) and (3.17) into equations (3.11) - (3.12), A_2, B_2 and δ_{cr} can be obtained from

$$\pi^2(2uA_2 + 2vB_2) - \delta\frac{\partial}{\partial A_2}\left(\int_0^1 F(\theta)dy\right) = 0, \tag{3.19}$$

$$\pi^2(2vA_2 + 8wB_2) - \delta\frac{\partial}{\partial B_2}\left(\int_0^1 F(\theta))dy\right) = 0, \tag{3.20}$$

$$\begin{aligned} &\left(2u\pi^2 - \delta\frac{\partial^2}{\partial A_2^2}\int_0^1 F(\theta)dy\right) \times \left(8\pi^2 w - \delta\frac{\partial^2}{\partial B_2^2}\int_0^1 F(\theta)dy\right) \\ &= \left(2\pi^2 v - \delta\frac{\partial^2}{\partial A_2 \partial B_2}\int_0^1 F(\theta)dy\right)^2. \end{aligned} \tag{3.21}$$

TABLE 1. Variation of δ_{cr} and θ_{cr} vs β for $G = 10$ for $N = 0$ and $\epsilon = 0$.

	$\Lambda = 0, \alpha = 0$		$\Lambda = 0, \alpha = 1$		$\Lambda = 1, \alpha = 0$		$\Lambda = 1, \alpha = 1$	
β	δ_{cr}	θ_{cr}	δ_{cr}	θ_{cr}	δ_{cr}	θ_{cr}	δ_{cr}	θ_{cr}
0.0	0.8784	1.1672	0.8784	1.1672	0.8784	1.1672	0.8784	1.1672
0.5	0.7605	1.3234	0.4532	1.9247	0.6770	1.4429	0.3986	2.0899
1.0	0.6788	1.4462	0.3314	2.2765	0.5673	1.6295	0.283	2.4796
1.5	0.6177	1.5481	0.2679	2.5136	0.4951	1.7728	0.2254	2.7363
2.0	0.5694	1.6356	0.2277	2.6939	0.4429	1.8898	.1897	2.9294
2.5	0.5302	1.7124	0.1995	2.8401	0.4029	1.9892	.165	3.0847
5.0	0.405	2.0011	0.1284	3.3241	0.2877	2.3422	.104	3.5934

TABLE 2. Variation of δ_{cr} and θ_{cr} vs β for $G = 10$ for $N = \infty$ and $\epsilon = 0$.

	$\Lambda = 0, \alpha = 0$		$\Lambda = 0, \alpha = 1$		$\Lambda = 1, \alpha = 0$		$\Lambda = 1, \alpha = 1$	
β	δ_{cr}	θ_{cr}	δ_{cr}	θ_{cr}	δ_{cr}	θ_{cr}	δ_{cr}	θ_{cr}
0.0	7.0187	1.1954	7.0187	1.1954	7.0187	1.1954	7.0187	1.1954
0.5	5.8944	1.3936	4.1595	1.7718	5.0461	1.537	4.0115	1.9809
1.0	5.1642	1.5431	3.1574	2.0727	4.0966	1.7516	3.007	2.3715
1.5	4.6379	1.6642	2.6035	2.2824	3.5071	1.9109	2.4619	2.6385
2.0	4.2344	1.7665	2.2414	2.4446	3.0954	2.0387	2.1089	2.8429
2.5	3.9119	1.8554	1.982	2.5776	2.7873	2.1457	1.858	3.009
5.0	2.9188	2.1827	1.3077	3.025	1.9325	2.5182	1.2129	3.5612

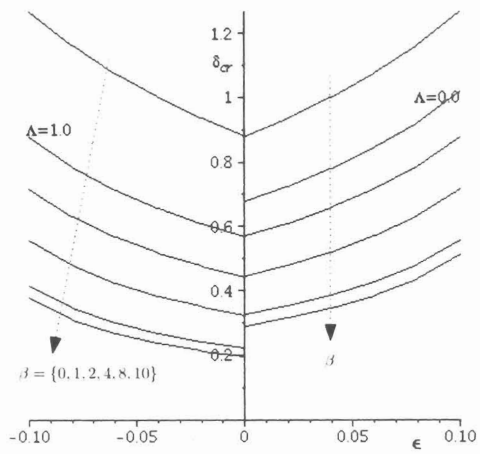

FIGURE 1. δ_{cr} vs ϵ for $G = 10$,$N = 0$,$\alpha = 0$,$m = 0.5$

4. Numerical Solutions And Results

The systems of equations (3.13)-(3.15) and (3.19)-(3.21) were solved simultaneously using the Mathematica 6 symbolic package by an interactive procedure. The definite integrals have been approximated by the Simpson's numerical integration scheme. To test the accuracy of the procedure, we compare our results with existing exact solutions in literature. For example, the particular case of $\beta = \epsilon = m = 0$, we observed from Tables 1 - 2, that in our computation, $\delta_{cr} = 0.8784$, while the existing exact solution $\delta_{cr} = 0.8785$[7, 18]. In addition, we obtained $\theta_{cr} = 1.1672$, while the existing exact solution $\theta_{cr} = 1.1870$. This shows that (δ_{cr}) is about 99.9%, while the critical temperature (θ_{cr}) is 98.3% close to exact solution. The transcendental appearance of θ in the Arrhenius expression may be responsible for the less accuracy of θ_{cr}.

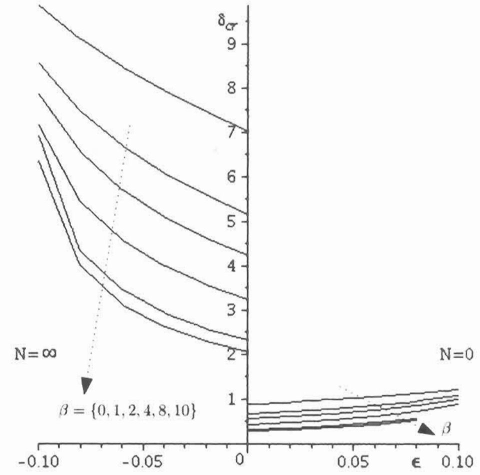

FIGURE 2. δ_{cr} vs ϵ, for $m = -2, \alpha = 0, G = 10, \Lambda = 0$

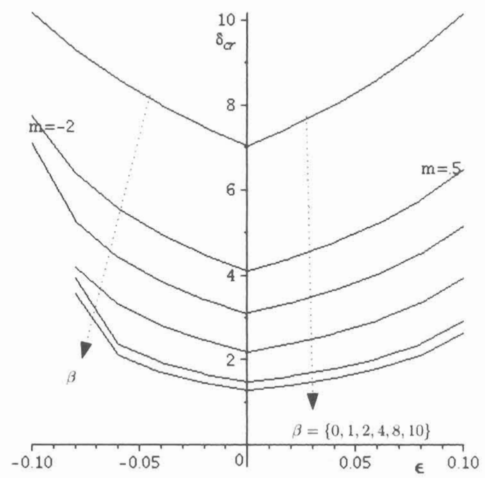

FIGURE 3. δ_{cr} vs ϵ,for $\Lambda = 1, G = 10, N = \infty$, $\alpha = 0$

Tables $1 - 2$ show that the Frank-Kamenetskii parameter (δ_{cr}) is monotonically decreasing with respect to the dissipation parameter(β), while the maximum temperature (θ_{cr}) is monotonically increasing with respect to β for $N = 0$ and $N = \infty$. Furthermore, the Couette flow configuration $\Lambda = 1$ attains ignition faster at a higher maximum temperature, when compared with the Poiseuille configuration $\Lambda = 0$. This may be due to additional frictional heat resulting from fluid/plate contact. In the adiabatic-Dirichlet configuration ($N = 0$), δ_{cr} and θ_{cr} are lower limits of the corresponding Dirichlet-Dirichlet($N = \infty$) configuration, thus making

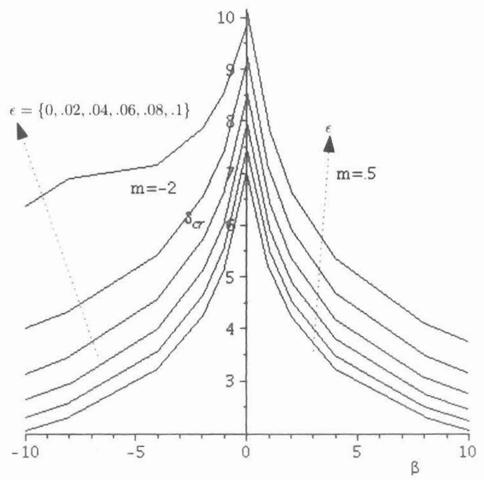

FIGURE 4. δ_{cr} vs β for $N = \infty, \alpha = 0, G = 10, \Lambda = 0$.

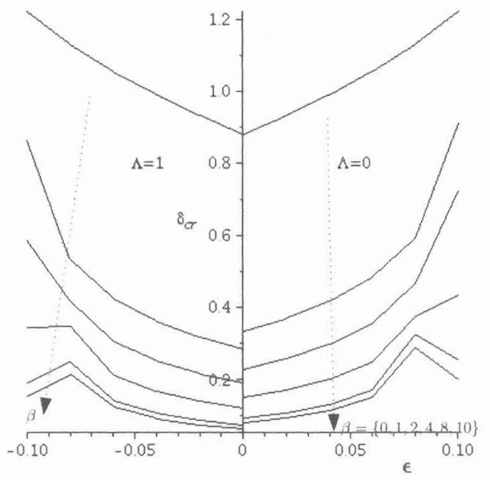

FIGURE 5. δ_{cr} vs ϵ for $G = 10, \alpha = 1, N = 0, m = -2$

thermal ignition faster in the previous than the latter. This may be due to the poor heat transfer due to an adiabatic lower wall($N = 0$), as compared with the perfect heat release($N = \infty$). Thus, the dissipation parameter, Couette flow and the adiabatic wall configuration lead to early occurrence of thermal ignition and would be desirable in applications requiring quick combustion.

The global parametric analysis of the system and the influence of the pre-exponential factor is made possible if the activation energy parameter is assumed different from zero ($\epsilon \neq 0$).

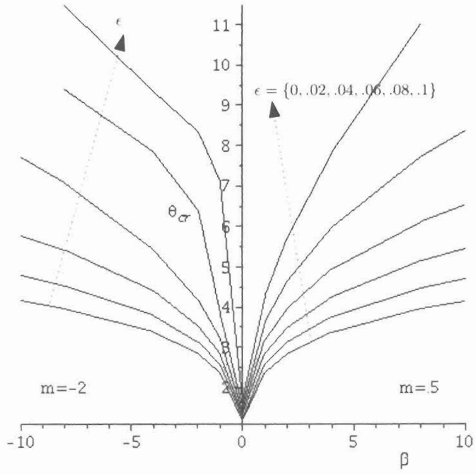

FIGURE 6. θ_{cr} vs β, $G = 10$,$N = \infty$,$\Lambda = \alpha = 1$.

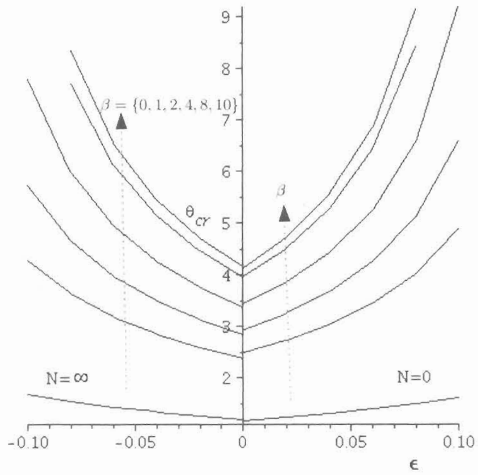

FIGURE 7. θ_{cr} vs ϵ for $\Lambda = 1$, $m = 0.5$,$G = 10$,$\alpha = 1$

Figs. 1 and 5 show the influence of Λ on the variation of δ_{cr} with respect to ϵ for $\alpha = 0$ and $\alpha = 1$ respectively for some β. In both cases, we observed that irrespective of the type of reaction(m), thermal ignition is faster in the Couette($\Lambda = 1$) than the Poiseuille($\Lambda = 0$) flow configuration. Figs. 2 and 8 show the influence of N on the variation of δ_{cr} and θ_{cr} with respect to ϵ for $\alpha = 0$ and $\alpha = 1$ respectively for some β. In Fig. 2, the case $N = 0$ shows that δ_{cr} is of a lower limit when compared with $N = \infty$. Fig. 8, shows that the maximum temperature θ_{cr} for $N = 0$ are also of lower than the case $N = \infty$ for some β. The behavior of δ_{cr} with

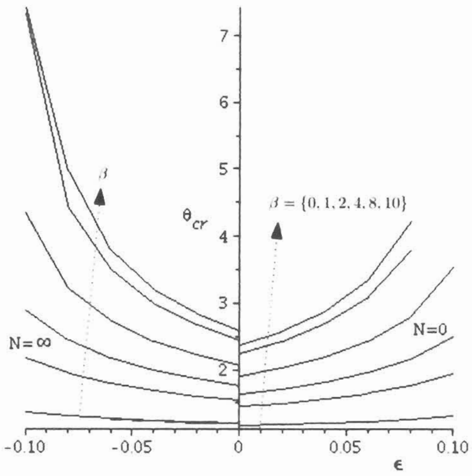

FIGURE 8. θ_{cr} vs ϵ for $G = 10$, $m = -2$,$\Lambda = 0$,$\alpha = 1$

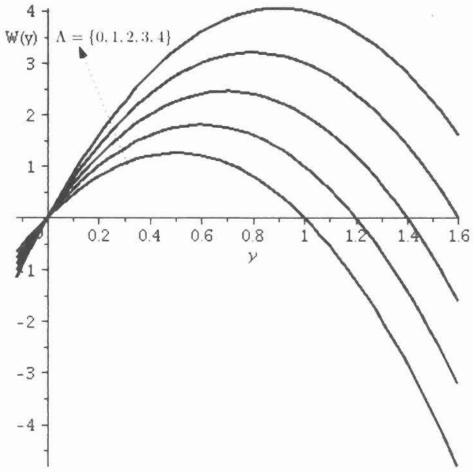

FIGURE 9. $W(y)$ vs y for $\alpha = 0$, $G = 10$.

respect to ϵ for two types of reactions: the sensitized($m = -2$) and Bimolecular($m = 0.5$) is shown in Fig. 3. Although, the difference is insignificant, δ_{cr} and θ_{cr} for the bimolecular case are lower than the sensitized case. This behavior is also observed for the ignition time in the homogeneous system(Ajadi and Okoya[4]). Fig. 4. shows that for the poiseuille flow($\Lambda = 0$), δ_{cr} is a decreasing function of β and that the case $m = 0.5$ are higher in values than the case of $m = -2$ for some ϵ. Fig. 7 shows the profiles of θ_{cr} for the Couette flow configuration($\Lambda = 1$)

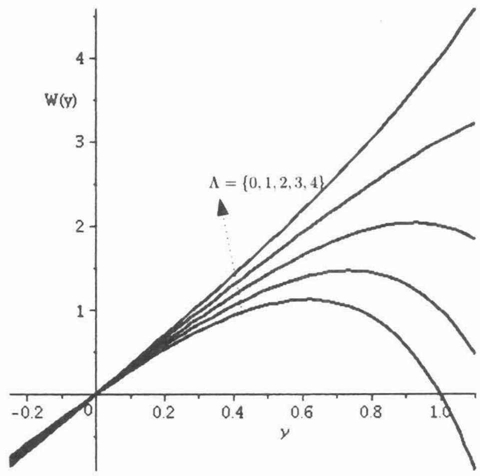

FIGURE 10. $W(y)$ vs y for $\alpha = 3$, $G = 10$.

for $N = 0$ and $N = \infty$ for some β. As stated earlier, θ_{cr} increases with β, while the adiabatic case($N = 0$) is an upper limit of the Dirichlet case($N = \infty$), which converges as β increases.

5. CONCLUSION

The study examined the contributions of wall dynamic and thermal conditions on the thermal ignition behavior of a one-step reaction system. It was shown that the Couette flow configuration leads to early occurrence of thermal ignition at a higher temperature than the Poiseuille flow. For the thermal conditions, adiabatic wall condition allows for early occurrence of ignition than the Dirichlet condition. In addition, viscous dissipation and variable pre-exponential have also lead to a departure from known results. This study is extremely important in the industries especially for safety and fluid effectiveness purposes among others.

6. ACKNOWLEDGMENTS

The author was supported by the Skirball postdoctoral fellowship of the Center of Advanced Studies in Mathematics at the Mathematics Department of Ben-Gurion University.

REFERENCES

[1] S.O. Ajadi: *A note on the thermal stability of a reactive non-Newtonian flow in a cylindrical pipe*, Int. Comm. Heat mass Transfer **36**(2009),63-68.

[2] S.O. Ajadi and V.Gol'dshtein: *Critical behaviour in a three step reactions kinetics model* Combustion Theory and Modelling **13**(2009),1-16.

[3] S.O. Ajadi: *Thermal Instability in a Model Chain Branching-Chain Breaking Kinetics*, MATCH Commun. Math. Comput. Chem. **53**(2005), 347-356.

[4] S.O. Ajadi and S. S. Okoya: *The Effect of Variable Pre - Exponential Factors on the Ignition Time of a Homogeneous System*, Int. Comm. Heat mass Transfer, **31**(2004), 143-150.

[5] S. Chen, Z. Liu, C. Zhang, Z. He, and Z. Tian: *A novel coupled lattice Boltzmann model for low Mach number combustion simulation*, Applied Mathematics and Computation **193**(2007), 266284.

[6] T. Fang: *A note on the incompressible couette flow with porous walls*, Int. Comm. Heat Mass Transfer,**31**(2004), 31-41.

[7] J.G. Graham - Eagle and G.C. Wake, *The theory of thermal explosions with simultaneous parallel reaction III. Disappearance of critical behaviour with one exothermic and one endothermic reaction*, Proc. R. Soc. Lond. A **407**(1986), 183 - 198.

[8] J.F. Griffiths and J.A. Barnardand, *Flame and Combustion, Blackie Academic and Professional*, Chapman and Hall(third edition), London, 1995.

[9] G.S.S. Ludford: *Combustion: basic equations and peculiar asymptotics*, Journal de Mecanique, **16**(1977), 531 - 551

[10] G.S.S. Ludford: *Reacting flows: Combustion and Chemical Reactors* (G. S. S. Ludford, ed), American Mathematical Society Publication, **24**(1986),129-161.

[11] O.D. Makinde: *Thermal criticality in viscous reactive flows through channels with a sliding wall: An exploitation of the Hermite-Pade' approximation method*, Mathematics and Computer Modelling **47**(2008), 312-317.

[12] O.D. Makinde: *Thermal Ignition in a reactive Viscous Flow Through a Channel Filled with a Porous Medium*, Journal of Heat Transfer, **128**(2006), 601-604.

[13] A. Majda and K.G. Lamb, *Simplified equations for low Mach number combustion with strong heat release*, in: Fife, P. C., A. Linan, F. A. Williams(Eds), Dynamical Issues in Combustion Theory, Springer, 1991.

[14] S.S. Okoya: *Thermal Stability for a Reactive Viscous Flow in a Slab*, Mechanics Research Communications **33**(2006), 728-733.

[15] S.S. Okoya: *Reactive-diffusive equation with variable pre-exponential factor*, Mechanics Research Communications **31**(2004), 263-267.

[16] S.S. Okoya and S. O. Ajadi: *Critical Parameters for Thermal Conduction Equations*, Mechanics Research Communications, **26**(1999), 363-370.

[17] J.R. Rodoia and J.F. Osterle: *Finite Difference Analysis of Plane Poiseuille and Couette flow developments*, Appl. Sci. Res., **10**(1960), 265-276.

[18] Ya.B. Zeldovich, G.I. Barenblatt, V.B. Librovich and G.M. Makhviladze, *The mathematical Theory of Combustion and Explosion*, Plenum Publishing Corp., New York, 1985.