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IGNITION TIME FOR A POROUS MEDIUM MODEL WITH LOW THERMAL CONDUCTIVITY

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ABSTRACT. The ignition time for a porous medium combustion arising from a porous medium combustion model is considered. The effect of the activation energy parameter as well as the parameter which measures the ratio of oxygen consumption to that of the solid(fuel), on the ignition time is presented.

1. INTRODUCTION

Porous medium (filteration) combustion is under intensive investigation due to its wide area of technical applications such as burning of coal, burninig of cigarettes, catalyst treatment by thermal waves, smouldering of polyurethane, etc. (see Wang [11] and Dobrego [2]). The temperature history of a one-dimensional combustion model arising from porous medium combustion

$$F + O_2 \qquad \text{heat} \qquad + CO_2 \tag{1}$$

is well documented in literature(see Okoya and Ajadi [8], Tam [13], Wang [11], Vartharajan and Williams [14], Norbury and Stuart [5]). Following Wang [11], the simplified heat equation model for (1) is given as

$$\frac{\partial T}{\partial t} = \frac{\partial}{\partial x}(k\frac{\partial T}{\partial x}) \; + \; \lambda f(T) = \frac{\partial}{\partial x}(k\frac{\partial T}{\partial x}) \; + \; \lambda(T)\frac{1+aT}{1+\exp(\frac{1-T}{\epsilon})} \tag{2}$$

$$T(\pm, t) = 0 \qquad \text{and} \qquad T(0) = 0$$

where T is the temperature of solid, $\lambda(T)$ is the pre-exponential factor, $f(T)$ is the reaction rate of the chemical, ϵ is the reciporcal activation energy, $a \geq 0$ is a parameter which determines the ratio of the rate of oxygen consumption to that of the solid, and k is the thermal conductivity. Wang [11] studied the bifurcation of positive solutions of a steady state problem arising from porous medium combustion model (2) for a constant pre-exponential factor. He obtained an explicit criteria for a unique soultions for $\lambda > 0$ when $a = 0$ and for an S-shaped bifurcation when $a \geq 0$. Tam [13] considered a problem in combustion theory with temperature dependent conductivity. they showed that information regarding criticality depaendence on data and parameter can be obtained from a transformed equation in which the conductivity is constant. Suceska [12] developed an own computer program named THERMEX for studying thermal ignition phenomenom. The program is tested with the result of calculations by other authors and a reasonable agreement was found under identical condition. Dobrego et al. [2] presented a two-dimensional, two-temperature and multi-component model for non -premixed filteration combustion simulation and a parametric study of radiative efficiency of the burner is performed.

Okoya [6] derived an analytic expression for the thermal ignition time of a reactive sysytem. They employed an effective activation energy appraoch which allows the elimination of previous difficulties arising in the estimation of the ignition time. Ignition time may be defined as the time of thermal runaway of the overall exothermic process. Thus it is of great importance to predict whether or not an explosive material will ignite under given conditions (see Varatharajan and William [14], Eckoff [3], and Martin et al. [4]). In the present work we study the system (2) assuming an adiabatic approximation, suggesting that the conduction processs is slow compared with the rate of heat release by the reaction. This is to facilitate a better understanding of the behaviour of the model.

2. MATHEMATICAL ANALYSIS

The adiabatic approximation of (2) reduces to an initial value problem

$$\frac{dT}{dt} = \lambda(T)\frac{1+aT}{1+\exp(\frac{1-T}{\epsilon})} \tag{3}$$

with the initial condition

$$T(0) = 0 \tag{4}$$

where the pre-exponential factor $\lambda(T)$ takes the form

$$\lambda(T) = T^m \tag{5}$$

and m is a numerical exponent. The variable pre-exponential form (5) has been exploited by previous authors (Boddington [1], Okoya [7]). For example $m = -2$ is the sensitised form, $m = 0$ is the Arrhenius whereas $m = 0.5$ is the bimolecular form. Furthermore, the case when $m \in \{-0.7, 0, 2.67\}$ has a rich history (see Sanchez et al. [10]). Thus the combination of (2)–(5) reduces to

$$t = \int_0^T \frac{1+\exp(\frac{1-u}{\epsilon})}{u^m(1+au)}du \ . \tag{6}$$

At the ignition time, $t = t_\infty$, $T \to \infty$, thus

$$t_\infty = \int_0^\infty \frac{1+\exp(\frac{1-u}{\epsilon})}{u^m(1+au)}du \tag{7}$$

By applying the Mathematica package to (7) we obtain expressions for the ignition times for $m = 0$, $m = -2$, and $m = 0.5$. Thus,

$m = 0$,

$$t_\infty = \frac{\exp(\frac{1+a}{a\epsilon})\Gamma[0, \frac{1}{a\epsilon}] + \log a}{a} \tag{8}$$

$m = -2$,

$$t_\infty = \frac{a\epsilon(a\epsilon - 1)\exp(\frac{1}{\epsilon}) + \exp(\frac{1+a\epsilon}{a\epsilon})\Gamma[0, \frac{1}{a\epsilon}] + \log a}{a^3} \tag{9}$$

$m = 0.5$,

$$t_\infty = \frac{3.14159 + 1.77245\exp(\frac{1+a}{a\epsilon})\Gamma[0.5, \frac{1}{a\epsilon}]}{\sqrt{a}} \tag{10}$$

Because (8)-(10) are not simple expressions, we further embark on a numerical scheme of Gauss Laguerre (n=14) in Mathematica for $m \in (-2, 0, 0.5)$ for equation (7). The following tables show the variation of the ignition time with the parameters ϵ and a for fixed m. The ignition time t_∞ is monotonically decreasing with respect to a for a fixed ϵ. Hence ignition time decreases with increasing ϵ. Since the increase in air (oxidant) supports combustion, this behaviour is physically reasonable.

Table 1. $\epsilon = .05$

a	0.5	1	2.0	3.0	4.0	5.0	6.0	7.0	10	100	1000
$t_\infty * 10^5 (m = -2)$	1.62	1.54	1.41	1.30	1.21	1.12	1.05	1.0	.85	.15	.17
$t_\infty * 10^5 (m = 0)$	161	154	141	13	121	112	106	99.5	84.6	15.4	1.68
$t_\infty * 10^5 (m = 0.5)$	509	487	446	411	383	357	335	315	268	48.8	5.31

Table 2. $\epsilon = 0.1$

a	0.5	1	2.0	3.0	4.0	5.0	6.0	7.0	10	100	1000
$t_\infty (m = -2)$	387	221	126	90	71	60	51.5	45	34	4.3	0.45
$t_\infty (m = 0)$	2040	1940	1770	1629	1510	1408	1319	1240	1053	191	21
$t_\infty (m = 0.5)$	6353	6054	5539	5108	4741	4423	4144	3900	3313	602	66

3. CONCLUSION

The tables describe the response of the ignition time (t_∞) to the dimensionless activation energy parameter as well the parameter for the ratio of air (oxidant) to fuel consumption (a). It could be seen that as a increases, t_∞ decreases monotonically for all m. For each a, t_∞ is a monotonically decreasing function of m. Thus, as $\epsilon \to 0$ (asymptotic limit), ignition is faster.

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