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NESTED BALANCED INCOMPLETE BLOCK DESIGNS OF THE DOUBLE CYCLIC TYPE

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ABSTRACT

A Nested Balanced Incomplete Block Design (NBIBD) is a design with two systems of blocks, each block from the first system containing m–blocks from the second. Consequently, by ignoring either system of blocks leaves a Balanced Incomplete Block Design (BIBD) whose blocks are those of the other system. Designs constructed in this paper, which are of the double cyclic type are possible if and only if, $v = b^1$, $k = r^1$ and $k = s^2$, where s is a prime or prime power. Otherwise the imposed relationship between the parameters of the designs would not be satisfied. In particular, double cyclic designs for $v = 7$ and two distinct double cyclic designs for $v = 13$ were constructed and presented in Table1.

KEYWORDS: Nested Design, Incomplete Block, Balanced Incomplete Block, Double cyclic.

INTRODUCTION

Blocking is the technique used to ensure homogeneity of experimental units within a sub-division of the experimental material, so that the treatment contrasts are estimated, making use of the intra-block information, with higher efficiency. For the experimental situations where there is only one nuisance factor, the block designs are used. When two such cross – classified factors are present, row – column designs such as Latin Square, lattice square, youden square, generalized youden, pseudo youden designs etc. are being used. In many fields and laboratory experiments the experimental units or conditions differ due to several factors which may influence the response under study. It might not always be possible to remove completely such heterogeneity due to the factors other than treatments by blocking alone. There are experimental situations in which there are one or more factors nested within the blocking factor. When there is one such factor, the resulting arrangement is regarded as nested block design.

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1. EXPERIMENTAL SETTINGS

Herein we shall describe three different experimental settings for the purpose of illustrating nested block designs.

1.1. This example relates to a biological experiment, quoted by Preece (1967). Suppose the half leaves of a plant form the experimental unit, on which a number of treatments, say inoculations with sap from tobacco plants infected with tobacco neurosis virus, are to be applied. Suppose the number of treatments is more than the number of suitable half-leaves for plant. Now, there is one source of variation present due to the variability among plants. Further, leaves within a plant may exhibit variation between themselves due to their being located on the upper branch, middle branch or on the lower branch of the same plant. Thus, leaves within plants form a nested "nuisance" factor, the nested being within the plants. The half-leaves being experimental units, we then have two systems of "blocks", leaves (which may be called sub-blocks) being nested within plants (which may be called blocks).

1.2. In experiments with animals, generally littermates (animals born in the same litter) are experimental units within a block i.e. litters are blocks. However, animals within the same litter may be varying in their initial body weight. If body weight is taken as another blocking factor, we have a system of nested blocks within a block.

1.3. Consider a field experiment conducted using a block design and harvesting is done block wise. To meet the objectives of the experiment, the harvested samples are to be analyzed for their content in the laboratory by different technicians at same time or by a technician over different periods of time. Therefore, to control the variation due to technicians or time periods, this is taken as another blocking factor, we have a system of nested (sub) blocks i.e. technicians or time periods within a block.

Note that for the experimental situations 1.1, 1.2 and 1.3 described above, we have one universe for which the results of the experiment will be valid. Out of this universe b_1 blocks of size k_1 have been selected and within each block, there are m sub-blocks such that sub-block size $k_2 = k_1/m$ and total number of the experimental units required is $b_1k_1 = b_1mk_2$.

2. NESTED FACTORS

In certain multi factor experiments, the levels of one factor (e.g. factor B) are similar but not identical for different levels of another factor (e.g. Factor A). Such arrangement is called a nested or hierarchical design with levels of factor B nested under the level of factor A.

Essentially, a nested factor has to do with how the levels of multiple factors are combined. A factor is said to be nested within a second factor if each level of the first factor occurs in conjunction with only one level of second factor. It is to be noted that nesting in general is not a symmetrical arrangement. For the purpose of illustrating nested designs we present below Figure 1.

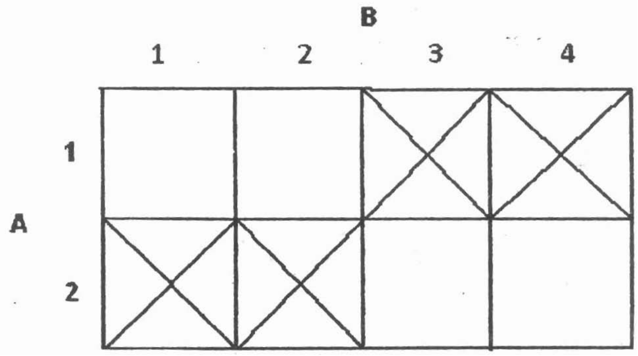

Figure 1. A simple nested design

The above Figure 1 illustrates a design in which factor B is nested within levels of factor A. Whereas level 1 of factor B occurs in conjunction with only level 1 of Factor A, level 1 of factor A occurs in conjunction with both level 1 and 2 of factor B. Hence, this is referring to as simple nested design. The above Figure 1 suggests one way of thinking about nested designs. It is that they are designs with missing cells.

More so, a factor can also be nested within multiple factors instead of a single factor. A factor is said to be nested within a combination of other factors if each of it levels occurs in conjunction with only one combination of levels of the other factors. Figure 2 below gives a clear illustration on this.

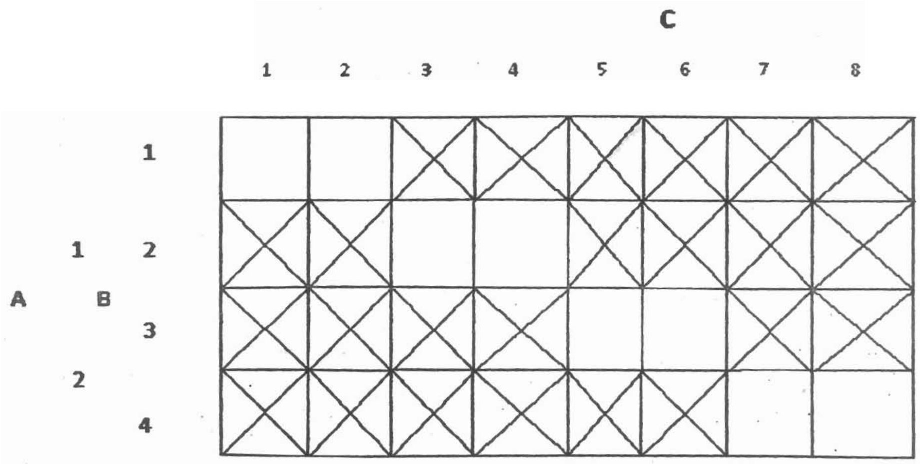

Figure 2. Nested within a combination of factors

Here, it could be seen that factors A and B have 2 levels each and are crossed with each other so that all possible combinations of levels of these two factors are represented. Factor C is indeed nested within these combinations of levels of factor A and B. Further, there are eight different levels of factor C. If C were to be crossed with A and B, there would be $2 \times 2 \times 8 = 32$ cells in the design. However, because of the nesting, we have observations in exactly eight cells.

3. CONCEPT OF NESTED BALANCED INCOMPLETE BLOCK DESIGNS (NBIBDs)

A NBIBD is a design with two systems of blocks, the second nested within the first (each block from the first system containing m–blocks from the second); such that ignoring either system leaves a Balanced Incomplete Block Design whose blocks are those of the other system.

Kleczkouski (1960) devised a form of Nested Incomplete Block Design with $v = 8$ treatments for a series of experiments in which bean plants, in the two primary leaves stage, were inoculated with sap from tobacco plants infected with tobacco necrosis virus. The treatments were eight different virus concentrations. Each leaf had two inoculations one for each half-leaf. Ignoring the leaf positions, plants and leaves were respectively, the block (of size 4) and sub-blocks (of size 2) of a nested balanced incomplete block Designs. Preece (1967) has for the case of two-way elimination of heterogeneity, one nested within the other introduced a Nested Balanced Incomplete Block (NBIB) design.

4. CONSTRUCTION OF NBIBDs FOR DOUBLE CYCLIC TYPE

Let there be a BIB design D_1 with parameters $v, b, r, K = s^2, \lambda$, where s is a prime or prime power. Consider another resolvable BIB Design D_2 with parameters; $= s^2$, $b = s(s+1)$, $k = s, r = s+1, \lambda = 1$. Now, take j^{th} block contents of D_1 as treatments and write a BIB design D_2 in these treatments and arrange the blocks of D_2 replication wise. Repeat this process for all blocks of D_1. This process results into NBIB design with parameters with group of blocks of D_2 forming a complete replicate is as block and blocks of D_2 within replications as sub-blocks (Rajender and Gupta,1993). Thus,

$$v = v^1, r = r^1(s+1), b_1 = (s+1)b^1, k_1 = s^2, b^2 = s(s+1)b^1, k_2 = s, \lambda_1 = (s+1)\lambda, \lambda_2 = 1 \quad (1)$$

Example 1: Let D_1 be a BIB design ($v^1 = 7 = b^1, r^1 = 4 = k^1 = 2^2, \lambda^1 = 2$), a solution of which can be obtained by developing the initial block (3, 5, 6, 7) mod 7. Let D_2 be the resolvable BIB design [$v^{11} = 4 = 2^2$, $b^{11} = 6 = 2(2+1)$, $r^{11} = 3 = 2+1$, $k^{11} = 2 = s$, $\lambda^{11} = 1$]. The contents of D_2 could be gotten using the procedure below.

Procedure/solution:

- Since $v = 7, k = 4$
- Parameters for unreduced BIBD are; $b = 35, r = 20, \lambda = 10$
- Parameters for reduced BIBD are; $b^1 = 7, r^1 = 4, \lambda^1 = 2$
- Now, using the knowledge of cyclic method and keeping treatment 7 in the initial block fixed. This eventually leads to the generation of two other replications.
- Also by further cyclic option on the initial two replicates, we then obtained the design below.

Let the initial block be [3, 5, 6, 7]

Replication-I		Replication-II		Replication-III	
[(3, 5),	(6, 7)]	[(5, 6),	(3, 7)]	[(6,3),	(5, 7)]
[(4, 6),	(7, 1)]	[(6, 7),	(4, 1)]	[(7, 4),	(6, 1)]
[(5, 7),	(1, 2)]	[(7, 1),	(5, 2)]	[(1, 5),	(7, 2)]
[(6, 1),	(2, 3)]	[(1, 2),	(6, 3)]	[(2, 6),	(1, 3)]
[(7, 2),	(3, 4)]	[(2, 3),	(7, 4)]	[(3, 7),	(2, 4)]
[(1, 3),	(4, 5)]	[(3, 4),	(1, 5)]	[(4, 1),	(3, 5)]
[(2, 4),	(5, 6)]	[(4, 5),	(2, 6)]	[(5, 2),	(4, 6)]

The parameters for the design in Example 1 are:

$v = 7, \quad r = 12, \quad b_1 = 21, \quad k_1 = 4, \quad \lambda_1 = 6,$

$b_2 = 42, \quad k_2 = 2, \quad \lambda_2 = 2$

***Example 2*:** NBIBD by Double cyclic for $v = 13, k = 2^2 = 4, r = 4, b = 13, \lambda = 1$

Solution: Let the initial block be [6, 10, 11, 13]

Replication-I		Replication-II		Replication-III	
[(6, 10),	(11, 13)]	[(10, 11),	(6, 13)]	[(11, 6),	(10, 13)]
[(7, 11),	(12, 1)]	[(11, 12),	(7, 1)]	[(12, 7),	(11, 1)]
[(8, 12),	(13, 2)]	[(12, 13),	(8, 2)]	[(13, 8),	(12, 2)]
[(9, 13),	(1, 3)]	[(13, 1),	(9, 3)]	[(1, 9),	(13, 3)]
[(10, 1),	(2, 4)]	[(1, 2),	(10, 4)]	[(2, 10),	(1, 4)]
[(11, 2),	(3, 5)]	[(2, 3),	(11, 5)]	[(3, 11),	(2, 5)]
[(12, 3),	(4, 6)]	[(3, 4),	(12, 6)]	[(4, 12),	(3, 6)]
[(13, 4),	(5, 7)]	[(4, 5),	(13, 7)]	[(5, 13),	(4, 7)]
[(1, 5),	(6, 8)]	[(5, 6),	(1, 8)]	[(6, 1),	(5, 8)]
[(2, 6),	(7, 9)]	[(6, 7),	(2, 9)]	[(7, 2),	(6, 9)]
[(3, 7),	(8, 10)]	[(7, 8),	(3, 10)]	[(8, 3),	(7, 10)]
[(4, 8),	(9, 11)]	[(8, 9),	(4, 11)]	[(9, 4),	(8, 11)]
[(5, 9),	(10, 12)]	[(9, 10),	(5, 12)]	[(10, 5),	(9, 12)]

The parameters of the design in Example 2 are:

$$v = 13, r = 12, b_1 = 39, k_1 = 4, \lambda_1 = 3,$$
$$b_2 = 78, k_2 = 2, \lambda_2 = 1$$

S/N	V	b_1	b_2	R	k_1	k_2	λ_1	λ_2	Initial Blocks
1.	7	21	42	12	4	2	6	2	[(3,5), (6,7)]
2.	13	39	78	12	4	2	3	1	[(6,10), (11,13)]
3.	13	104	312	72	9	3	48	4	[(3,5,6), (7,8,9), (11,12,13)]

Table 1. Table of Nested Balanced Incomplete Block Designs, $4 \leq v \leq 13$ and $12 \leq r \leq 72$

***Key*: Round brackets ( ) are used for blocks of size k_2 (sub-blocks) while square brackets [ ] are used for blocks of size k_1 (main blocks).**

CONCLUSIONS

From the summary of the designs constructed in this paper, given in Table 1, it is evident that double cyclic method is applicable if and only if, $v = b^1, k = r^1$ and $k = s^2$(where s is a prime or prime power), otherwise the imposed relationship between the parameters of the designs would not be satisfied. Also, from Table 1 it is evident that the method of Double Cyclic for NBIB designs has a very limited list.

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