RESULTS OF FIXED POINTS FOR SOME CONTRACTIVE MAPS IN BANACH SPACES

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A B S T R A C T

This study investigated the approximation of fixed points in relation to the notion of approximate operator and their various contractive maps in Banach spaces on Picard, Mann and Ishikawa iterations with the aim of establishing new results, extending and generalizing some of the existing results in literature. This study also investigated some new convergence conditions with the aim of getting some entirely new convergence results on Picard, Mann and Isikawa iterations in Banach spaces.

The notions of approximate operator, continuity and monotonicity of functions, concepts of α-contractions, comparison functions, Zamfirescu mappings, asymptotically non-expansive mappings, asymptotically demicontractive mappings and multi-valued weakly Picard mappings were all employed. Some properties of metric spaces, normed Iinear spaces and Banach spaces were also used. In some of the methods of this study, $'T'$ was assumed to be a selfmap of a Banach space E while U w a s assumed to be an approximate operator of T. The estimate $\|p - q\|$ was computed, where p is a fixed point of T and q is an assumed fixed point of U. The following contractive definition

$$\|T x - Ty\| \leq 2\delta \| x - T x \| + \delta \|x - y\| , \qquad \forall x , y \in E \qquad (i)$$

was also used, where E is a normed linear space and T is a selfmap of E satisfying the conditions of a Zamfirescu mapping and $0 \leq \delta < 1$. Some new parameters were introduced and the following contractive definitions

$$d(Tx,Ty) \leq \delta[d(x ,Tx) + d(x,Ty) + d(x , y)], \qquad \forall x , y \in K \qquad (ii)$$

and

$$d(Tx,Ty) + d(x , Tx) + d(y , Ty) \leq 2\delta d(x, Ty) + \theta d(x,y), \quad \forall x , y \in K \qquad (iii)$$

were used, where K is a closed and convex subset of a complete metric space E and T is a selfmap of K with $2 \leq \theta < 5$ and for some suitable $\delta \in [0, 1]$. The following φ–contractive-type definition

$$\|Tx - Ty\| \leq \varphi(\|x - y\|) + L\|x - Tx\|, \qquad \forall x, y \in E \qquad (iv)$$

was also used, *where $L \geq 0$ is a constant,* φ $: \mathfrak{R}_+ \to \mathfrak{R}_+$ is a comparison function, E is a normed linear space and T is a selfmap of E. Modified iteration schemes with uniformly continuous asymptotically nonexpansive and uniformly continuous asymptotically demi-contractive mappings were also employed.

New fixed points and convergence results were established for Picard, Mann and Ishikawa iterations in Banach spaces in the sense that for a suitable $\eta > 0$ small enough, we have

$$\|p - q\| \leq \frac{2\delta}{(1-\delta)} \|x_n - p\| + \frac{(2\delta + 1)\eta}{(1-\delta)}, \quad 0 \leq \delta < 1, \quad n = 0, 1, 2, \ldots \quad (v)$$

where p is a filed point of T and q is a fixed point of U. Some new convergence results were also obtained on the modified schemes in Hilbert spaces. The equivalence between the convergences of lshikawa, Mann and Picard iterations in Banach spaces was also established. The notion of weak contraction from the case of single-valued mappings to the case of multi-valued weak φ—contractions was also extended.

In conclusion, this study generalized and extended some existing fixed points and convergence results in literature as well as the equivalence between the convergences of Ishakwa, Mann and Picard iterations in Banach spaces. This study also extended the notion of weak contraction from the case of single-valued mappings to the case of multi-valued weak φ— contractions which is an improvement over the existing results in literature.