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THE MANY 'FACES' OF THERMAL EXPLOSION: MATHEMATICAL MODELLING POINT OF VIEW

By

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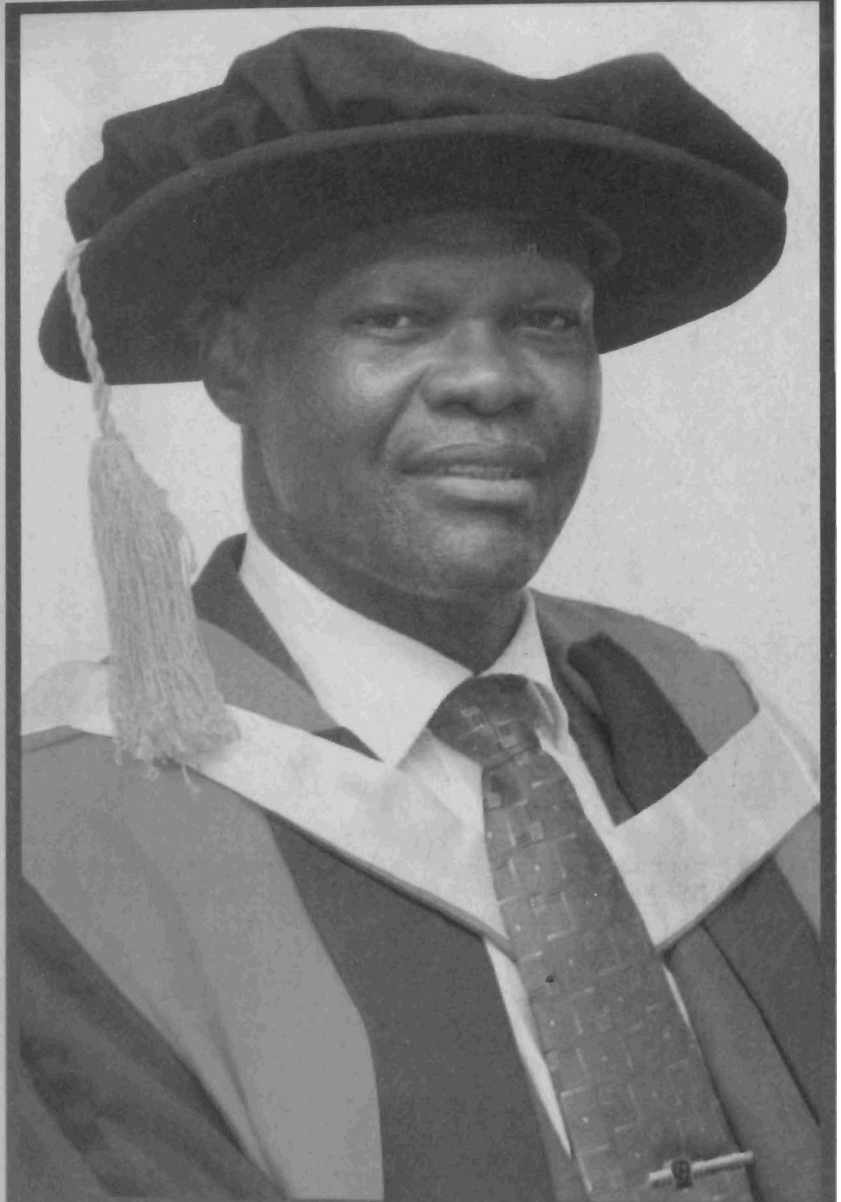

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An Inaugural lecture Delivered at Oduduwa Hall
Obafemi Awolowo University, Ile-Ife, Nigeria
On Tuesday 13th November, 2018.

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PROTOCOL

The Vice – Chancellor, Professor Eyitope Ogunbodede; The Deputy Vice Chancellor (Academic), Professor Adebayo Bamire; The Deputy Vice Chancellor (Administration), Professor Chris Ajila; The Registrar, Mrs Margaret Omosule and other Principal Officers of the University; Members of Senate of the Obafemi Awolowo University; Provosts; The Dean, Faculty of Science, Professor (Mrs) Olubukola Adedeji; Deans of other Faculties; Directors; Heads of Departments; Esteemed Colleagues; Royal Fathers; Ministers of the Gospel; Distinguished Alumni here seated; Distinguished Guests; my Precious Students, past and present; Gentlemen of the Press; Esteemed Online Audience; Distinguished Ladies and Gentlemen.

I would first of all like to thank the Vice – Chancellor Professor Eyitope Ogunbodede for chairing this lecture, and to thank the University for the opportunity to deliver this public lecture on the subject of combustion. I am grateful to the University authority for making this important event possible that has helped me reflect on my journey so far as a Mathematician (*i.e. where I have been, where I am now and how I can effect changes in my career path in the future*). I particularly welcome those who have come a distance to be with us this evening – people from Akure, Ondo, Osogbo, Ibadan, Abeokuta, Lagos, Oyo, Ilorin, Abuja and elsewhere. Thank you for taking the time and effort to travel down, your presence is highly appreciated.

PREAMBLE

It is with deep appreciation and gratitude to the Almighty God that I mount this podium this evening to deliver the 329th Inaugural Lecture of the Obafemi Awolowo University, on a topic that relates closely to events we experience, perhaps daily, entitled **The many 'Faces' of Thermal Explosion: Mathematical Modelling point of view**.

It is important to note, Mr. Vice Chancellor, Sir, that after 34 years of meritorious service and due consideration of my stewardship in the academic profession, I deem it fit to deliver my inaugural lecture, the first in the field of applied (applicable) mathematics.

For those of you who are not friendly with Mathematics as a subject, this inaugural lecture is supposed to be comprehensible to the educated layperson. So, rest assured, even if your knowledge of Applied Mathematics is sketchy, or missing in large parts, I shall not attempt to bore you with too much Mathematics but to make you view mathematics as a conceptual framework for organizing and trying to understand the world better. In addition, I will use this unique academic tradition as a yardstick for academic excellence and career relevance as a Professor as well as sustained commitment to knowledge and research.

In retrospect, I had, as teachers and mentors, some remarkable Mathematicians who made the highest expressions and the abstractness of mathematical thinking visible and simplified the ambiguity in the subject from my primary school through to secondary and at the higher institutions. I will dwell more on the higher institutional experience.

I entered the sphere and rigour of mathematical training via my first exposure to advanced mathematics at the University of Ibadan Extra-moral evening classes. More so, as a freshman at the University of Ife (now Obafemi Awolowo University, OAU), I was thrilled to discover vectors and applied mathematics through my amiable and dedicated teacher, retired Professor E. A. Akinrelere during the 1979/80 academic session. He later taught my set courses in mechanics as well as in fluid dynamics which are components of Applied Mathematics. It was his presentation in the teaching of these subject areas that made me pursue a research career in applicable mathematics. I finished my undergraduate degree at University of Ife and earned a B.Sc (First Class Honours) in 1983. I proceeded to the M.Sc. programme in this same University with the encouragement and support of Professor R. O. Ayeni (who passed on to glory in 2012) and completed same in

1986. I happened to be the first doctoral student of Professor R. O. Ayeni, and completed my Ph.D. degree in 1989 from Obafemi Awolowo University.

I started my teaching career at the Department of Mathematics, then University of Ife (now OAU) as a Graduate Assistant in 1984, while late Professor Ayeni was the Acting Head of the Department and rose through the ranks to become a Professor of Mathematics in 2003. Ten years later, in June 2013, I was appointed the first occupier of Pastor E. A. Adeboye Outstanding Professor of Mathematics (Professorial Chair of Mathematics) endowed at the University of Lagos and returned to OAU in June 2017 after four meritorious years of service to humanity.

As I progress in this lecture, I will point to my accomplishment as a Mathematician in my thirty four years and forty four days in service to this great citadel of learning (OAU).

WHY LEARN MATHEMATICS

The issue of learning is all about the stages of our educational development; primary, secondary and higher institution. At the primary level, we are introduced to small faltering steps such as sum or addition (with the sign +), subtraction (-) and multiplication (x) which live most naturally in Arithmetic. What is more is that these wonderful signs are the bedrock that built the future for Science, Technology, Engineering and Mathematics (STEM). The new experience to the pupil is exciting and it is supposed to be the child's launching pad. Once the foundation is properly laid at this level, there is no need to go over the basic teachings about signs again at the secondary level. In the secondary school, we are taught to focus on developing mathematical equations from everyday living, laws of nature, trigonometric and other functions, differentiation and integration as well as mild abstraction. At the University, these ideas are made more concrete and expanded into many facets with emphasis on limiting possible errors. It is the University's role to ensure that these students contribute to society through knowledge acquired and pursue research at the highest levels. In fact, mathematics permeates both everyday life and numerous academic and professional endeavours. It can be termed the language of science and technology.

However, when mathematics is applied to the real world, the most important thing is not whether it represents the complex mathematical techniques, but does it tell something meaningful about the reality? We make an impact by solving important mathematical problems derived from everyday living under realistic physical conditions.

Mr. Vice Chancellor Sir, I must say that I decided to continue my study in mathematics after my first degree in this University partially due to its practical application to real life and the fact that I enjoy the logical way mathematical arguments are structured which exposes one to prompt thinking and in-depth reasoning.

WHO ARE MATHEMATICIANS?

Mathematicians lie somewhere at the intersection between artists and writers on one hand and physicists, chemists and biologists on the other. As applied Mathematicians, we try to construct mathematical problems from the physical world based on development and our understanding of nature. Mathematicians research into useful related problems in STEM or even in resolving concerns in social sciences.

Let me share from the quotes of some Mathematicians extracted from the literature:

> ***"It was a period of intense work- searching for clues in what had been done, trying and retrying ideas until I could force them to take shape – a period of frustration, too, but punctuated by sudden thrilling insights that encourages me to think I was on the right track. .... Mathematics has been studied by mankind for thousands of years. Rulers have come and gone, countries have come and gone, and empires have come and gone. But through it all and surviving the wars and the plaques and the famines is the thread of Mathematics. It is one of the few constants of human life. The Mathematics of the ancient Greeks and Chinese dynasties is as valid now as it was then. Mathematics will continue also into the future. The unsolved problems of today will be the solutions of tomorrow's world. I feel extremely fortunate to be part of this long and fascinating story"*** Sir Andrew John Wiles, Department of Mathematics, University of Oxford, United Kingdom (UK).

> ***"I am sometimes asked, especially by students, how I go about deciding what topics or problems to work on. This is a hard question for me to answer precisely, some Mathematicians explicitly decide that they want to solve a particular problem, or they set out to develop a large research problem in a certain area. I do not work in this way. In my case, what tends to happen is that I find that I am curious about certain things at any given time and I want to understand them better. (Sometimes these are things that are well understood by some other people, but not by me.) I go to talks and seminars, I read papers, I talk to people and I ask myself questions, I play around with examples, and in this way one thing leads to another and new ideas emerge. I should also point out, however, that most of the ideas that I have and the things that I try do not work, and I suspect that the same is probably true of many other Mathematicians. Of course this means that perseverance is a process, and that it is very important not to just give up too easily."*** Professor Adebisi Agboola, Department of Mathematics, University of California at Santa Barbara, United States of American (USA).

Mr Vice Chancellor sir, I want to put on record that I actually belong to the same school of thought of the above quotes.

My mission in this lecture is therefore to attempt to reveal to you and this great audience here today that mathematics has always played a role and will continue to play a leading role in the development of science and technology as well as in social and economical advancement and to show us that it is very significant in addressing those issues that confront us in our daily lives. my main objective in this lecture is to address the question: Is mathematics the answer to the problems we face today and the solution to the challenges we are yet to realize?

One beautiful thing about Mathematics, as a science subject is that it does not require very expensive equipments or hundreds of assistants. However, more often than not, one is up alone against the unknown or with a team of collaborators, succeeding or failing on your own wit. Fortunately, it is easier to handle a pen than a sword. Handling a pen in this manner has given me the opportunities to be inspired and to inspire others.

Just imagining being a mathematician is a wonderful thing, but experiencing it is far more interesting. In my experience, mathematics is like secret garden or special places where one could grow exotic and beautiful theories. You need a key to get in, a key that you earn by letting mathematical structures revolve in your head until they be real as the room you sit in or the environment where you dwell. The framework of mathematical modelling in applied mathematics is synonymous to solving word problems in secondary schools. At its foundation lies a physical problem motivated by nature under certain conditions. The physical descriptions of the problem are then converted to mathematical arguments in form of equations (such as differential or algebraic equations) called the model equations. These are solved, validated against related experiments and interpreted for relevant use.

In the following, I shall try to summarise some of my research interests and activities within the last thirty four years, during this period I was predominantly concerned with closed–form solutions, functional–analytical and computational approaches to (system of) ordinary and partial differential equations (ODEs and PDEs). Rigorous physical arguments with extensive mathematical results

provide important evidence for my research work. In my opinion, functional framework combined with many other mathematical subjects, is well suited for providing powerful and sufficient insight in the study of nonlinear differential equations.

Clearly, I shall only be able to scratch the surface, and most of the time I shall be rather vague. Nevertheless, I hope that I can give an idea of "what is behind the scene" to both the specialist and non-specialist in this great audience.

WHAT IS COMBUSTION?

Combustion literarily means "burning". Combustion in a device is a striking concept which has played a central role in research and development in processes involving explosion, ignition, spark, flame etc. In this lecture, I shall describe how a variety of physical systems and mathematical models demonstrate the phenomenon of combustion. Evidently, any substance that can burn is classified as

This lecture will focus on fossil fuels such as coal, crude oil and natural gas. Let me remind or inform this audience that the mixture of fuel (combustible material) and oxygen in a chemical reaction produces energy in the form of heat, light and sometimes but usually accompanied with waste products in the form of gases and/ or by products. Symbolically, this process is summarized as

$$\text{Fuel} + \text{Oxygen} \rightarrow \text{Waste} + \text{Energy}. \qquad (1)$$

This system (1) is termed a combustion reaction.
It turns out that when a hydrocarbons (say coal) reacts with oxygen (air) it produces carbon dioxide, ash and energy simultaneously i.e.

$$\text{Coal} + \text{Oxygen} \rightarrow \text{Carbondioxide} + \text{Ash} + \text{Energy}. \qquad (2)$$

In this scenario, the waste products of the chemical reaction are carbon-dioxide and ash. We come to a second example of chemical hydrogen and oxygen which produces water as the waste product while the desired product is energy

Hydrogen + Oxygen → Water + Energy. (3)

It is a well-known fact that air or specifically oxygen is necessary for combustion and there are credible proofs in literature that it is a necessary reactant in combustion. Having presented the simplified combustion, the heat present in the system is measured variations in temperature which is expressed according to a comparative scale and revealed by a thermometer or perceived by touch.

Another feature one might mention is that combustion reactions are divided into two broad categories namely slow and fast reactions. Let me single out one reaction for special investigation. At room temperature and atmospheric pressure, hydrogen and oxygen hardly react over a period of many years. On the other hand, if you mix hydrogen and oxygen gases and heat source is applied then a fast reaction takes place in a split second leading to a violent shattering or blowing apart of container. This act is called explosion or simply ignition rather than burning which entails flames. The same phenomenon happens in a combustion engine (e.g. in cars). When fuel is mixed with air in the cylinder of the engine of the car it explodes with sparks from the spark plug. With this occurrence in a combustion engine there is the need to develop more efficient, but cleaner, engines and fuels for the automobile and oil industries. Several combustion reactions are very fast and we shall mention them later. Another important form of combustion reactions which occur without the aid of initiating devices are termed spontaneous explosion. Spontaneous explosions are of two types. They may be due to the exothermicity of the reaction (thermal explosion) or its chain character (chain explosion). These two types of spontaneous explosions will be discussed further in a while. There is no need, I hope as evident from the title of this lecture, to discuss the slow reaction

The combustion reactions discussed above have positive and important influence in the way we live in the society. In our environment, we need the same reactions to move cars, ships and

aeroplanes, to heat water, to cook, for manufacturing and to produce electricity and maybe you know of some too.

There is another related area of usefulness of combustion reactions that are explosive. In fact, we can use explosives to blast rocks which stand as an obstacle during road constructions or when trying to get access to underground waters, to destroy buildings that have defects which constitute a nuisance in communities and in war times explosives are freely used in the battle ground to destroy the opponents. These are 'Intentional explosion' and proving effective in every day usage.

Given all of the rich benefits of combustion reactions, there still seems to be quite a lot of scenarios which sometimes prove unhelpful and can always get out of control (unhappy ending). Sometimes the pain and despair is just too much. Examples abound in Nigeria and around the globe of unintentional explosions producing a great deal of damage in the society: *"Gas Plant Explosion Kills up To 100 People in Anambra State"* was *reported by Sahara Reporters, New York, 24th December, 2015. It was reported in part that "Up to 100 people are believed to have perished in a gas plant explosion in Nnewi, Anambra State today. The Christmas Eve disaster reportedly began at 11am when a fire ignited as a commercial truck was discharging its contents at the Inter Corp Oil Ltd gas plant."* Huge fire ball was seen during the explosion.

A more recent fatal accident was reported in the Daily Post of Nigeria and published on 1st July 2018: *"Otedola Bridge fire: Lagos government reveals cause of explosion, announces restriction for tankers." It reads in part "The Lagos State Government has released initial findings into the fatal accident that happened on the Otedola Bridge, near the Lagos-Ibadan expressway on Thursday June 28, 2018. The accident caused by a petrol tanker, led to the death of nine people, while over 50 cars were also burnt.* " Emergency crews were invited to the screen to save lives and the awful thing about this explosion is not only that it devastates but it also constitute environmental health hazard.

More recently, 9NEWS reported at 8.18 pm on 26th July, 2018 that *furnace has exploded at an iron foundry in Adelaide sending workers fleeing". It was claimed that "Dozens of workers have been forced to run for their lives after a furnace exploded in an iron foundry in Adelaide's northern suburbs causing $1 ... ... of damage. One worker suffered minor burns to his head when the blaze broke out at the McKechnie foundry at Gepps Cross shortly after 9am. 'They had four furnaces with molten ... in them at about 1500 degrees Celsius and two of those furnaces had exploded,' Alan Fisk from the Metropolitan Fire Service said."*

Also, a common occurrence during the harmattan season in our locality is the ignition of dry grass from sparks (say from an unextinguished cigarette stuck) leading to an outbreak of fire. The winds quickly extend the blaze, burning through a field where the value of properties put at risk is inestimable. It can take few units from many fire departments over an hour to curtail such blaze. The scenes of such event normally reveal several acres of land and its content left charred. Several wild fires have been reported this year alone, In fact, there were recent cases of wild fire in California, USA in early August 2018.

In discussing this unwanted combustion, it is evident that forests, buildings, equipments, household gas cylinders and even clothing can act as fuels. Therefore, fire-prevention scientists have to ensure that these materials do not cause devastating harm on people by understanding the mechanisms through which they combust and providing safety measure in handling them. I must commend the directive of the Federal Government to all institutions to ensure the protection of life and properties through the creation of fire service stations. OAU Fire Service Station is situated about 250 meters from this hall.

It is worth noting that in this lecture we are interested in nuclear chemicals which explode with a rapid release of energy and in due course we shall mention some powerful explosives that

are not nuclear weapons. It is not in the scope of this lecture to spell out how easy it is to make non-nuclear explosives commonly used in improvised explosive devices (IEDs). This was done to discourage terror attacks such as being witnessed in the country under Boko Haram.

In the light of the preceding paragraphs, Williams (1986) writes that, "*Combustion represents the science of exothermic chemical reactions in flows with heat and mass transfer*". This definition has the advantage of identifying the disciplines underlining the subject – thermodynamics, chemical kinetics, fluid dynamics and transport processes". These four items may be reorganised in conservation laws as follow: Conservation (of mass, momentum, energy and chemical species) as well as associated sources.

It is worthy of note that the theory of combustion of various fuels, ... hydrocarbons, in air or oxygen-inert mixtures has been developed. The rate of the reaction varies extremely rapidly with temperature. The formula for the rate of reaction resulting from theoretical consideration of Arrhenius (1889) named after him is $k = A\exp(-E/(RT))$ (here A is the pre- exponential multiplier, E the activation energy of the combustible mixture, R the universal constant, T the temperature). This expression serves nowadays as a basic analytical tool for most of the theoreticians working in the field of combustion and other closely related fields of chemical engineering. However, this expression is a source of high nonlinearity in combustion models. In order to summarize and give a broader view to the topic under consideration, I shall now provide some typical expressions that approximates and generalizes the Arrhenius form of temperature dependence of a reaction rate constant for a one step reaction (Boddington et al., 1981, Boddington et al. 1983b, Boddington et al., 1984 as indicated in the following Table:

Table 1: The tabular representation of experimental and approximate results showing the various rate coefficients from

literature where $\theta = (T - T_a)/(RT_a^2/E)$, T_a is the ambient or initial temperature and β is the activation energy parameter.

	Rate Coefficient k	Non–dimensional form		Name
a	$A\exp(-E/RT)$	$A\exp\left(-\frac{E}{RT_0}\right)f(\theta)$	$f = \exp(\theta)$ $= \nu^5(1+\gamma\theta)^5$ $= \theta^2 + (e-2)\theta + 1$	Exponential approximation Quintic Approximation Quadratic approximation
b	$AT^n\exp(-E/RT)$	$A\exp\left(-\frac{E}{RT_0}\right)f(\theta)$	$= (1+\beta\theta)^n \exp\left(\frac{\theta}{1+\beta\theta}\right)$	n = ½, Bimolecular Kinetics n = 0, Arrhenius Kinetics n = -2, Sensitized Kinetics

TYPES OF CHEMICAL EXPLOSIONS IN COMBUSTION

There are many types of chemical explosions that have been firmly established in the science of combustion but I have studied two distinct types. Therefore, due to space and time may I indulge the specialist amongst us to please concur. In late 1920s, experimental proofs were advanced for the concepts of thermal and chain branching explosion with evidence that these essentially distinct types of explosion actually occurred. Firstly, in thermal explosion the overall exothermic reaction increases the temperature of the reacting gas, thereby accelerating the rate of reaction and thus the rate of heat released from the system. As the reaction proceeds, the reaction rate remains immeasurably small up to some critical value, which depends on the specific experimental conditions. With subsequent build up of heat, even few degrees of temperature above critical, the combustible mixture reacts very rapidly producing a great deal of damage unless heat is rapidly and sufficiently abstracted from the system so that the resulting reaction is a 'fizzle'. It is worth noting that a standard parameter of interest in the conduction theory of thermal explosion is the Frank-Kamenetskii parameter which is a measure of the strength of

the reaction mechanism (See, for example, Frank-Kamenetskii – 1969 for the classical case of $\beta = 0$).

Secondly, a chain reaction has been defined as a succession of reactions where a reactive product or by-product causes further reactions to occur. Particularly, Soviet Physicist in the person of Nikolay Semenov in 1934 carried out a quantitative chain chemical reaction theory and the main steps in chain reaction are as follows:

(a) Initiation (formation of free radicals or initiating devices);
(b) Chain branching (a propagation step which forms more radicals);
(c) Chain breaking reactions or Termination (elementary step in which the active radical loses its activity);

Chemical reactions are capable of reacting with chain mechanism but with the added feature that each cycle of the chain produces a net increase in the number of radicals is of great importance in combustion theory. In the event that it represents a chain branching step with say 7 radicals formed per cycle, in 100 cycles of such a chain, each original radical would have grown to 7^{100} (Kapila, 1978). It is this Malthusian growth of branching which makes it possible for it to invoke an explosion. This is called branched-chain explosion.

The increase in the concentration of the radicals is, of course, slowed down by the termination steps which occur either through three-body collisions in the interior or at the wall of the vessel. The competition between the branching rate and the termination rate determines the eventual outcome of the reaction.

In addition, there are explosion processes that exhibit characteristics of both branched-chain and thermal explosions with or without initiation. The classical problem of the concept of a branched-chain thermal explosion was formulated and solved under some assumptions in Melentiev and Todes, (1941). Examples are the well-known reduced-chemistry description of hydrogen-oxygen system and acetylene-oxygen ignition. For such reactant combinations, a good definition for the ignition time is the time to thermal runaway, from a local hot spot, in the overall exothermic chain-branching step. Interest therefore exists in deriving ignition-time expressions as well as criticality for a general model chemistry of branched-chain thermal explosions. In practice, when initiation reaction is not important, chemical reactions can still occur with the aid of initiating devices such as an electrical spark, a hot wire or a subsidiary flame. It is worth noting that in the classical formulation by Semenov (1959) in the limit of large activation energy approximation ($\beta \longrightarrow 0$), thermal explosion for exothermic chemical reactions with heat loss (assuming Newton's law of heat exchange) was first discussed analytically. This practically important Semenov problem is independent of the geometry. He defined criticality as the point at which the second derivative of the heat balance equation is equal to zero. The emerging parameter was named after him as Semenov number. Physically it means that a quiet, nonexplosive proceeding of the reaction is possible only if the Semenov number is less than a critical value. This condition is known as the condition of thermal explosion. In the event that $\beta \neq 0$, (but $\beta < \beta_{tr}$) there are two critical values (a maximum and a minimum) for the Semenov number representing ignition and extinction, but subsequently there is no extreme at $\beta = \beta_{tr}$, (i.e. disappearance of criticality or transition). The branched-chain thermal explosion theory shows consistency with the thermal explosion theory of Frank-Kamenetskii.

A popular view of the reaction of hydrocarbon and air is that it forms a one step reaction as an overall combustion (see for example reactions (1-3)) but there is a lot more to it than that. In fact, it has been experimentally deduced that reaction (3) occur in more than 50 elementary steps; see Semenov (1959), while the chemical – kinetic description for acetylene oxidation involves as much as 613 reversible steps among more than 100 chemical species, just to mention a few. With a target precision of all reaction rates, it is clear that there will be need for huge mathematical equations to handle this reality.

To turn the combustion phenomena into workable tool, one needs mathematical modelling in order to appreciate the mathematical practices.

MATHEMATICAL MODELLING

The role of mathematical model may be defined as an idealization possessing some of the features of reality, but not all. Invariably, one needs to select only those that play key roles in the reaction for inspection. Once the mathematical model is in place one needs to make quantitative predictions of what will occur in the domain of definition (closed or open vessel).

Since joining the academic service of the Department of Mathematics of the then University of Ife (now Obafemi Awolowo University) and during my days as a student, I have been fascinated by the fact that mathematical models under realistic assumptions can help to disclose hidden information about a physical system and lead to a wide variety of differing physical responses which, at a first glance, seemed strange and beyond our immediate understanding.

Most mathematical models of combustion involve nonlinear differential equations, which are largely coupled systems of partial or ordinary differential equations (PDEs or ODEs). In order to give a somewhat broader view, I now present some of the interrelationships of my research interest with the theory of PDEs.

Firstly, for the investigation of nonlinear differential equations it is important to have surveyed the available technicalities in order to arrive at the heart of the matter. Very often the problem is reduced to investigating qualitative properties such as proving the existence, uniqueness, boundedness and stability of solutions of the PDEs. The experience often led to developing tools which not only help to solve the given problem, but provides a platform to tackle long standing problems. Secondly, the nonlinear problems for some unknown function may be solved exactly but in most cases the problems cannot be resolved exactly. The solutions of these

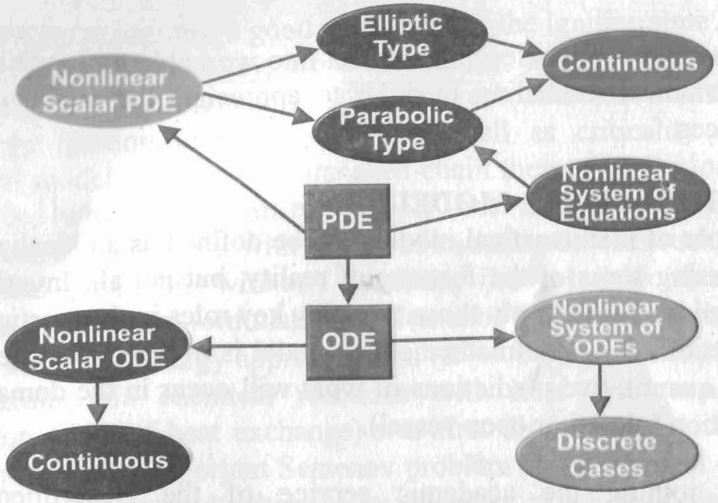

Figure 1: Broader view of the inter -relationships in the theory of PDEs and ODEs.

nonlinear PDEs are dominated by their stationary or moving singularities: physically, a real singularity controls the local behaviour of a solution. The schematic diagram below shows the rough categories of tools which I have employed in the course of my research work.

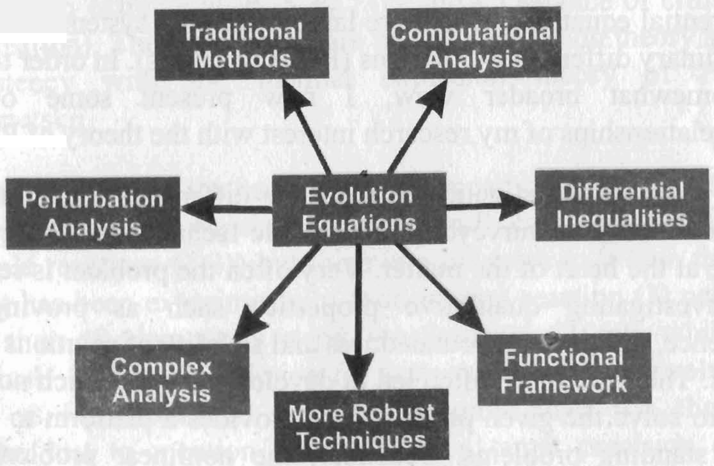

Figure 2: A schematic diagram of the tools employed in my research work.

Of course, mathematical methods have contributed to the progress of the science of combustion in a number of ways. Mathematics began to play a role in the subject at the turn of the 20^{th} century, but not until the Second World War did mathematical studies in combustion intensified. A notable exponential growth occurred in applications of mathematics to combustion during the 1970's as exemplified by a recent monograph on the subject by Williams (1986).

To pursue this study it is necessary to define thermal explosion from the mathematical point of view. One notes that the appearance of thermal explosion corresponds to the disappearance of solution. Mathematically, this concept may manifest in two broad forms: first is the appearance of unboundedness (singularity) in the solution in finite time while the second manifestation is bifurcation phenomena (criticality) of nonlinear equations in the form of multiplicity of solutions. The results in my research work represent a composite of calculations by both methods. The first method is applicable when the shape of the reaction vessel is not relevant while the second takes into cognisance the shape of the vessel. Conditions for singularity and/or criticality, therefore, depend on the relative importance of the temperature – dependence of reaction rate.

Mr. Vice Chancellor Sir, it seems appropriate to bring a description of the phenomenon to the audience with a background in mathematical modelling and without specialization in combustion theory.

BASIC EQUATIONS

As a basis for further discussion I begin with a model problem to illustrate the origin of some of the equations of combustion and to point out the variables.

Let Ω be a bounded domain in $\mathbb{R}^n$, where n =1, 2, or 3 in most physical relevant situation, with a smooth boundary Γ and outer unit normal η. For non mathematical audience, figure 3 gives a pictorial example that we meet in our everyday living.

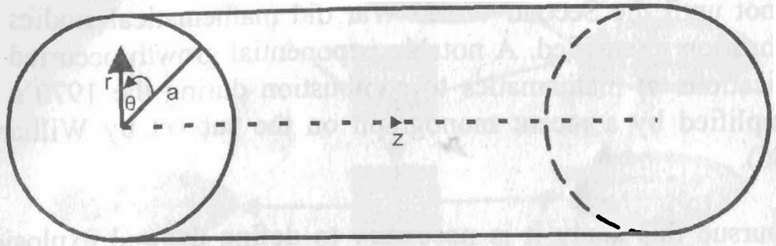

Figure 3: The physical model of Ω for 3-dimensional container (horizontal circular cylinder) with coordinate system (r, θ, z).

We further assume that Ω is occupied by some species (say a chemical substance, a population, heat, etc.) with which we can associate a space and time dependent quantity U. Also suppose that that it can move/ or change within Ω. We further assume that the distribution of the species throughout Ω at a given time, say at $t = 0$, is described by the "initial quantity" U_0 and that one wants to predict this distribution at any given later time t. The basic hypothesis which we impose is the law of conservation of mass, i.e. Fick's law. If we denote f as the 'production quantity', in general the resulting differential equation is of parabolic type subject to appropriate boundary conditions such as cases of physical interest: Dirichlet (fixed value) $U = a$, Neumann (isoflux) $\frac{\partial U}{\partial \eta} = a$ or Robin (radiative) $\frac{\partial U}{\partial \eta} + bu = a$. This leads to the following initial boundary value problem

$$\begin{aligned} &\frac{\partial U}{\partial t} - \nabla \cdot (K(U,\mathrm{x})\nabla U) = f \quad \text{in } \Omega \times (0,\infty), \\ &\frac{\partial U}{\partial \eta} = g \quad \text{on } \Gamma \times (0,\infty), \\ &U(\cdot,0) = u_0 \quad \text{on } \Omega. \end{aligned} \tag{4}$$

Equation (4) is a reaction-diffusion equation of a scalar unknown. In the particular case where $K = 1$, equation (4) reduces to the initial value problem for the heat equation. In fact, the general case

of the differential equation in (4) is of parabolic type. It is worth noting that much research is presently devoted to the investigation of problems of this type.

In most mathematical models of relevant physical (chemical, biological, sociological, etc.) systems one has to consider involving several (say N) species moving within Ω and interacting with each other. The basic hypotheses in this case are the assumptions that with each specie n we can associate a quantity U_n and that the law of conservation of mass holds for each n. Thus we arrive at the system of N conservation laws (see Amann 2003)

$$\begin{aligned} &\partial U_n/\partial t - \nabla \cdot \bar{J}_n = f_n \quad \text{in } \Omega \times (0,\infty), \\ &-\eta \cdot \bar{J}_n = g_n \quad \text{on } \Gamma \times (0,\infty), \\ &U_n(\cdot,0) = u_{0n} \quad \text{on } \Omega, \end{aligned} \tag{5}$$

for $1 \le n \le N$. In general, the flux vector $\bar{J}_n$ as well as the exterior densities f_n and g_n of species n may depend on U_n and all other species as well, that is, on the full vector $U = (U_1, \ldots\ldots, U_N)$. Here a natural generalization of the constitutive law states that the flux vector $\bar{J}_n$ depends linearly on the gradient ∇U_n for $1 \le n \le N$, that is

$$\bar{J}_n(U) = -a_{n1}\nabla U_1 - \cdots\cdots - a_{nN}\nabla U_N, \quad 1 \le n \le N, \tag{6}$$

where, in general, the diffusion coefficients a_{ni} depend nonlinearly on u. In many applications this hypothesis can be justified on the basis of thermodynamical considerations (De Groot and Mazur, 1984). In fact, for the case of $N = 2$, we obtain the following system of reaction diffusion equations:

$$\begin{aligned} &\partial U_1/\partial t - \nabla \cdot (a(U_1)\nabla U_1 + b(U_2)\nabla U_2) = f_1(U_1, U_2) \text{ in } \Omega \times (0,\infty), \\ &\partial U_2/\partial t - \nabla \cdot (c(U_1)\nabla U_1 + d(U_2)\nabla U_2) = f_2(U_1, U_2) \text{ in } \Omega \times (0,\infty), \end{aligned} \tag{7}$$

subject to appropriate boundary and initial conditions (Kirane, 1989). Thus the reaction-diffusion system (7) is a particular instant of a general parabolic initial boundary value problem.

It is worth noting that equation (4) or (5) and (b) is augmented with the usual hydrodynamic variables (density, velocity, pressure and gravity) arising from conservation of mass and momentum when the species is in motion. The fundamental equations describing the dynamics of the system are conservation of the mass and momentum:

$$\nabla \cdot \bar{V} = 0,$$

$$\rho \frac{D\bar{V}}{Dt} = \rho\left(\frac{\partial \bar{V}}{\partial t} + \bar{V} \cdot \nabla \bar{V}\right) = \nabla \cdot \mathbf{T} + \bar{b}, \quad (8)$$

where, $\bar{V}$ denotes the velocity, $\bar{b}$ is the body force, ρ is the density, D/Dt denotes the material time differentiation and $\mathbf{T} = \mathbf{T}(p, \dot{\gamma}; \mu)$ is the Cauchy stress tensor in which $\dot{\gamma}$ is the shear rate, p is the pressure and μ is the dynamic shear viscosity. Cauchy stress tensor, **T**, for the third-grade fluids can be written in the following form

$$\mathbf{T} = = p\mathbf{I} + \mu\mathbf{A}_1 + \alpha_1\mathbf{A}_2 + \alpha_2\mathbf{A}_1^2 + \beta_3(\mathrm{tr}\,\mathbf{A}_1^2)\mathbf{A}_1, \quad (9)$$

in which p is the pressure, tr is the trace of a unit matrix, **I** is the unit tensor, μ is the dynamic shear viscosity. Here α_1, α_2 and β_3 are the material coefficients which are generally of the temperature variables but assumed to be constant in this lecture. The kinematic tensor $\mathbf{A}_1$ and $\mathbf{A}_2$ as well as thermodynamical compatibility can be found in Massodi and Christe (1995), Chinyoka and Makinde (2012) and Adesanya and Falade (2015)

This hydrodynamic concept brings me to another important fluid property namely the dynamic shear viscosity. The form of the dynamical viscosity models employed in my research work are displayed in Table 2. Even though these examples are standard and depend on temperature or the spatial coordinates, there are often surprisingly other forms that depend on pressure or concentration.

Another physical property of interest is the thermal conductivity. The thermal conductivity of the fluids is much less sensitive to temperature changes than the viscosity. Tolerably accurate fitting models for thermal conductivities are provided in Table 3.

Table 2: Viscosity trends for fluids

	Viscosity μ	Name
a	μ_0	Constant viscosity
b	$\mu_0 \exp(-m(T - T_0))$	Reynold's viscosity model
c	$\mu_0 \exp\left(\frac{A_0}{B_0 + T} - T_w\right)$	Vogel's viscosity model
d	$\frac{\mu_0}{1 + m(T - T_0)}$	Approximate of Reynold's viscosity model
e	$\mu_0 \exp\left(\frac{C_o}{T^{q_0}}\right)$	Special type of oil or Barus viscosity model
f	r^{q_1}	Power-law spatially distributed model

Here, T_0 is the initial or ambient temperature of the system, T_w is the wall temperature, μ_0 is the reference viscosity, m, q_0, A_0, B_0 and C_0 are constants.

Table 3: Thermal conductivity trends for fluids

	Thermal conductivity K(T)	Name
i	$K_\infty\left(1 + \epsilon\frac{T - T_\infty}{T_w - T_\infty}\right)$	Linear type
ii	$K_0\, T^s$	Power-law type
iii	$K_0 \exp(\alpha_1 T)$	Arrhenius type
iv	$K_0 \frac{\exp(-\alpha_2/T)}{T^\kappa}$	Generalized thermal conductivity

Here K_∞ (or T_∞) is the thermal conductivity (or temperature) of the fluid far away from the surface, K_0 is the initial thermal conductivity of the fluid, T_w is the wall temperature s is a numerical index and $\alpha_i, i = 1, 2$ are constants.

In order to make realistic and useful predictions of combustion reactions from the mathematical models, it is advisable to provide closed – form (or analytical) solutions. In the event that closed – form (or analytical) solutions are not available, we resolve to numerical computations. It is possible to also employ tools in estimating and understanding (qualitative properties) the solutions. The development of the theory is now complete and the ground rules for compact presentations of analytical and / or numerical attacks on the problems are clear. Accordingly, in the next section, I shall present the results from my research activities since the inception of my career.

MY RESEARCH FOCUS AND CONTRIBUTIONS TO COMBUSTION THEORY

Mr. Vice Chancellor Sir, I shall now attempt to give a very brief highlight of some of my research focus and contributions to modelling of combustion and flow of reactive fluids.

My research is a broad, delightful and rare blend of the theoretical, the computational and the applied.

My main contributions are largely along 5 main areas of Applied Mathematics: reaction-diffusion systems, spatially homogenous reactions, scalar reaction-diffusion equations, Reactive system, flows of reactive non-Newtonian fluids, flows of nonreactive Newtonian fluids and non-Newtonian fluid. Fortunately, some eminent Nigerian researchers such as: Professor E. A. Akinrelere, Late Professor R. O. Ayeni, Late Distinguished Emeritus Prof. Vincent Olusegun Olunloyo and Professor David Olarewaju Olagunju now relocated to Department of Mathematical Sciences, University of Delaware, Newark, USA and a host of others; had earlier carried out theoretical studies, and reported along their findings along these lines. The contribution to combustion theory in Nigeria continues today, beyond this presentation and the work of my students and colleagues.

My research efforts received a boost from numerous grants I had obtained and my international collaboration fostered by several international organizations such as International Mathematical Union (IMU) Commission for Exchange and Development, Abdus Salam International Centre for Theoretical Physics (ICTP), Trieste Italy, International Centre for Mathematical Sciences (ICMS), Edinburgh, United Kingdom, Max-Planck Institute for Mathematics (MPIM) in the Sciences, Leipzig. Germany, International Centre for Mechanical Sciences (CISM), Udine, Italy, Vienna University of Technology, Vienna, Austria, Third World Academy of Sciences, Trieste, Italy and Institute of Mathematics Its Applications (IMA), University of Minnesota, Minneapolis, USA. These mathematics communities have been fully welcoming and supportive.

REACTION-DIFFUSION SYSTEMS

My journey in the study of reaction-diffusion systems started with my personal favourite mathematical problem which concerns the size of the parabolic system (7) when $b = 0$ (Okoya, 1994) as an extension to the results in an earlier work (Kirane, 1989). The equations model the behaviour of temperature and the reactant specie when diffusion of both properties is allowed but restricted to a one-step reaction. This problem arises in chemical reactor theory which obeys the general Arrhenius rate law under various initial and boundary conditions. Conditions for local and global existence of solutions in appropriate Banach space were provided using general theory of semi group and maximum principle for parabolic equations. Two lemmas and one main theorem were postulated and proved by established upper growth bound for the component U_2 as well as for the other component U_1 subject to the Neumann-Neumann boundary conditions with bounded Arrhenius rate law and the diffusion coefficients $a > d$ by employing integral transformations, supremum norm as well as differential inequalities. Results for the homogenous and non-homogeous Dirichlet-Dirichlet problems were also presented. In addition, one of the first significant contributions within the broad area of pure mathematics in the article is that I provided convincing arguments for the first time on the conditions for parabolicity of the PDE. Moreover, the article presented a mathematical framework for the system of parabolic equations which provides relevant qualitative information. Review on the coupled reaction-diffusion equations revealed that "***the mathematical arguments are laid out clearly... and it is a paper much more for specialists in analysis than*** *specialists in combustion theory*"- Unknown referee. Consequently, I was encouraged to put physical perspective into my subsequent publications.

During the same year, the effects of additional reaction step as a model of chain branching and chain breaking (termination) kinetics involving concentration of fuel, concentration of radicals and temperature were determined (Okoya and Ayeni, 1994). For this special case, $N = 3$ in equation (7), with Neumann conditions while modelling consecutive reactions were derived from elementary steps for hydrogen combustion. A theorem was stated and the tools employed in the mathematical analysis of the proof include maximum principle for parabolic equations, the notion of upper and lower solutions as well as comparison arguments. Specifically, the study showed that small variations in initial data induce only small errors in the solution.

I also studied a particular model in the treatment of the two step reaction (Okoya, 1997):

$$A \longrightarrow B \longrightarrow C. \qquad (10)$$

The first reaction, $A \longrightarrow B$ in equation (10) will convert the given unburned state to a partially burned state. This produces a flame with a certain speed. The second reaction, $B \longrightarrow C$ in (10) will then convert this partially burned state to a completely burned state. This also produces a second flame. The above flames may propagate, each corresponding to a different stage in the reaction, and proceeding with a different velocity. One can therefore imagine two flames both existing, but the distance between them ever increasing (Buckmaster and Ludford, 1982). An in-depth mathematical analysis was carried out for the case $N = 3$ and results show that boundedness were obtained for Dirichlet, Neumann and Robin boundary conditions for the three-component reaction-diffusion equation system. Extensive analysis revealed that if the same type of boundary conditions are imposed on U_2 and U_3, boundedness of solutions follows regardless of the type of boundary conditions that is imposed on U_1. Interesting limiting cases were shown in the paper and compared with those in the

literature. Results are in excellent agreement with those in the literature. However, the situation where boundary conditions of different types, not considered above, are imposed on U_1, U_2 and U_3 presents fundamental difficulties. It was concluded from the results that the presence of diffusion in the reaction system does not cause loss of boundedness properties for virtually non-negative initial data subject to various boundary conditions of uniform and the above mentioned mixed types.

Our mathematical studies (Okoya, 1994a, Okoya and Ayeni, 1994b and Okoya 1997) revealed that under careful experimental conditions, each system under investigation is safe from mishap.

SPATIALLY HOMOGENOUS REACTIONS

A problem in chemical reactor theory in which the conduction process is slow relative to the rate of heat released by reaction process of the chemical specie can be regarded as spatially homogeneous (see Dainton 1966 [p 103]). In this context the chemical reaction is well stirred in an open system and is formally represented by ordinary differential equation(s) involving the usual Arrhenius term(s) (i.e. the diffusion coefficients $a_{ni} = 0$ in equation (5) and (6)).

Characteristics of homogeneous explosions in homogeneous, adiabatic systems have been elucidated in (Okoya, 2001a and 2002a). The earlier paper described an investigation of the explosion time for an extended model involving four step mechanism based on some features of hydrogen oxidation. Significant deviations from past results were obtained leading to fresh determinations of criticality of an emerging parameter and explosion times for which the results of my calculations agree well with those obtained previously. The boundary between the explosion and slow temperature region was found. In the later case, we presented an asymptotic analysis of the model equations and based on the new formulation, it was shown that the structure of the solutions is richer than those in the previous results. A theorem was stated and proved while a comparison of the quintic time of explosion with the computed value reveals that the quintic time of explosion is a lower solution. These theoretical analyses of the explosion times are the safe period (before explosion) under experimental conditions.

A related paper by me (Okoya, 1999) contains the proposed modified effective activation energy (EAE) which allows the elimination of previous limitations in computation of thermal ignition time from the integral representation for the combustion of shocked nitromethane. Also, with my then Ph.D student and young collaborator who accompanied me to Abdus Salam ICTP (Ajadi and Okoya, 2004), we studied the ignition time of a system of homogeneous three step reaction mechanism. The variation of the pre-exponential factor led to significant departure from the Arrhenius case which was published. This research helped to show for the special case of shocked nitromethane that the numerical and new analytical solutions are lower limits of the EAE, thus providing an extension to this line of research.

In these cases, mathematical analysis in the research group here in Ile - Ife has shown that one can attain explosion (unboundedness) in finite time if the values obtained for some emerging parameters are sufficiently attained in the laboratory. Thereby these studies ensure safety of storage.

Another specific area of contribution of applied mathematics to the progress of combustion theory is in the calculation of evolution of species concentration during homogeneous, isothermal, chemical-kinetic processes with multiple-step chemistry, and the criteria for the applicability of steady-state approximations (Williams, 1986). In what follows, explosion processes that exhibit characteristics of both branched-chain and thermal explosions shall be investigated. Two considered examples are the highly reduced-chemistry description of hydrogen-oxygen system and acetylene-oxygen mixture with an initiation rate taking to be constant. Secondly we consider chain branching reaction with strong temperature-dependent branching rates as well as exothermic branching and

termination of chains. For the chemical process addressed, energy conservation in the adiabatic system results in

$$\text{Temperature} = \text{Ambient Temperature} = \alpha_0\,[\text{H}], \qquad (11)$$

where the constant α_0 denotes the energy released in the process per mole of fuel consumed, divided by the product of the density and specific heat at constant pressure for the reactant mixture and [H] denotes concentration of the active species, H.

In one of my publications (Okoya, 2006a), I extended the study of Varantharajan and William (2000 and 2001) and investigated the effect of heat exchange between the system and its surrounding. Mostly analytical investigations of the simplified model using standard Semenov's techniques were presented. The analytical method provides expressions for criticality and the transition points. With collaborators in Ladoke Akintola University of Technology, Ogbomoso (Ayeni et al., 2003), we revisited the ignition times earlier studied in (Varatherajan and Williams, 2000) with constant initiation rate and accounted for generalized heat loss during ignition regime. Numerical investigations were performed for particular values of the index and the results were displayed graphically. The contribution shows how severe the heat loss could be to prevent ignition from occurring. The existence of the general power law heat loss term explains another mathematical model of the branched chain thermal reaction as contained in Okoya (2006b). Employing the Semenov technique I generalized the results in Okoya (2006a). Next, when linear time dependent initiation rate to accommodate a small-scale leakage (conductive loss) that may occur through a hole of a broken apparatus was modelled, it resulted in a stiff nonlinear ordinary differential equation (Okoya, 2009); I further showed numerically the different qualitative effects of varying the dimensionless parameters. These results give substantial new information about the ignition, explosion, and detonation of real reactant mixtures.

So far we have only considered a purely kinetic approximation without diffusion. We now considered another extreme case when radical depletion is negligible. Highly exothermic systems such as gaseous ditertiary-butyl peroxide, methyl nitrate and shocked nitromethane are most practical examples readily at hand. These reactive mixtures, with large sizes, higher ambient temperatures or densities lead to an initial period of slow temperature development and reactant consumption which is then followed by an extremely rapid acceleration of the rate. During this, the temperature excess rises to a high value almost instantaneously. At the same time, the concentration of the reactant falls virtually to zero. Following the sharp "ignition" pulse, the temperature excess decays exponentially to zero (see Boddington et al., 1983a). The mathematical description of the latter stage is readily apparent when $N = 1$ in equation 5 (temperature equation only).

SCALAR REACTION - DIFFUSION EQUATIONS

When a coal stockpile is stored in the presence of air, slow oxidation of the carbonaceous materials occurs and heat is released. If the rate of heat generation from local hot spots within the stockpile is greater than the rate of heat dissipation and transportation to the external environment, the self – heating of the coal stockpile ensues. The self – heating of coal stockpiles has a long history of posing significant problems to coal producers because it lowers the quality of coal and may result in hazardous thermal runaway. Precise prediction of the self – heating process is, therefore, necessary in order to identify and evaluate control measures and strategies for safe coal mining, storage and transportation. Such a prediction requires accurate estimate of the various processes associated with the self – heating, which are impossible unless the appropriate phenomenological coefficients are known. In such storage – type problems, the critical ignition temperature $\theta_{\max\,cr}$, also known as the critical storage temperature, is an important design and control parameter, since at higher temperature than $\theta_{\max\,cr}$, thermal ignition occurs, possibly giving rise to a variety of instabilities and other associated problems. As indicated by (Boddington et al., 1977), '**Even when reactions are kinematically simple and obey the Arrhenius equation, the differential equations for the heat balance and reactant consumption cannot be solved explicitly to express temperatures and concentrations as functions of time unless strong simplifications are made.**' One of such simplification is to assume that there is no reactant consumption and that is the approach that we have taken in the succeeding paragraph.

CASE 1: Unsteady

Here the steady state thermal explosion in a material undergoing a zero-order exothermic reaction with the reactant consumption ignored, and with the outer surface of the slab held at the ambient temperature T_a (Frank-Kamenetskii boundary conditions) in one and two dimensional cases was studied. Mathematically, general system under discussion with variable thermal conductivity can be reduced exactly (Frank – Kamenetskii, 1969) to the equations

$$\frac{\partial T}{\partial t} = \frac{1}{x^i}\frac{\partial}{\partial x}\left(x^i K(T)\frac{\partial T}{\partial x}\right) + \delta\frac{Q(T)}{x^\varkappa}, \qquad (12)$$

with appropriate initial and boundary conditions where $i = 0, 1, 2$ correspond to infinite slab, infinite cylinder and sphere, respectively and $\varkappa$ takes on integral values.

In 1996 I studied the steady state material (equation (12)) for both power law dependency on thermal conductivity (K) and source term (Q). I exploited simple mathematical procedure to obtain new exact solutions for the infinite slab, infinite cylinder and sphere for some interesting special cases. In particular, these exact solutions admit combustion phenomena such as unique non-vanishing real solutions, multiple solutions and explosion for the co-axial cylindrical case.

The need to develop more exact solutions, equation (12) was studied by Okoya and Ajadi (1999) for two physically relevant models when both thermal conductivity (K) and source term (Q) are power-law as well as exponential forms. Studies were carried out for certain combustion processes between two infinite cylindrical materials undergoing exothermic reaction. By prescribing physically relevant thermal conditions, we successfully obtain numerically the critical values of a non – dimensional Frank-Kamenetskii parameter δ as a function of a certain emerging constant.

Despite the great importance and recorded success in studying the steady state limits, as in most models, however the unsteady state problem addresses a more realistic scenario. In this wise, the paper Okoya (2001b) investigates two mathematical models resulting in heat transfer problem by utilizing the concepts of self-similarity and phase – plane methods. The study showed that for spatial polynomial decay in the reaction term there is a very rich range of simple similarity solutions with a wide range of differing physical response. The behaviour include blow-up, extinction, moving singularities and moving free boundaries. Some of the solutions were illustrated graphically.

Simple exact solutions obtained in the paper (Okoya 2002b) for an arbitrary number of dimensions are direct products of suitable transformations and standard techniques. The presentation of original research results for unsteady and steady states of the non-linear diffusion equation apply to numerous models which have hitherto not been studied. The solutions exhibit uniqueness, multiplicity and the phenomenon of singularity, criticality, oscillation, extinction and blow-up. Some of these phenomena were depicted graphically.

I was further motivated to study a diffusion-reaction equation (Okoya, 2005) for the concentration of one species in one-dimensional domain and assuming the diffusion coefficient and the source term are of special power-law. Closed-form solutions are found in phase space in terms of elementary functions for some special parameter combinations.

Okoya et al., (2010) examined the linear heat equation with convection and presented six closed-form solutions of a special type of separation of variables by functional framework. The results were presented in the form of a theorem. In the limiting

case, the stated theorem guarantees existence of seven solutions for the classical heat equation.

CASE 2: Steady

The corresponding steady – state with constant thermal conductivity for thermal reaction in dimensionless form is

$$\Delta\theta + \delta f(\theta) = 0 \quad \text{in } \Omega, \tag{13}$$

with the Newtonian cooling on the boundary

$$d\theta/d\rho + \alpha\theta = 0 \quad \text{on } \partial\Omega, \tag{14}$$

Here $\Delta = (1/\rho^j)d/d\rho(\rho^j d/d\rho)$ is the Laplacian operator, Ω is the volume of the reactant and $\partial\Omega$ is the surface of the reactant, $\rho = r/a$, is the spatial variable expressing the distance from the centre of the body for either ($j = 0$) a slab with thickness 2 units and infinite length, ($j = 1$) a cylinder with radius unity and infinite, or ($j = 2$) a sphere of radius unity. Thus j specifies the dimension of the body in which the reaction takes place. In the limiting case in which the Biot number tends to infinity, the boundary condition at the surface becomes

$$\theta = 0 \quad \text{on } \partial\Omega. \tag{15}$$

(Frank-kamanetskii boundary condition).

NATURAL CONDITIONS FOR CRITICALITY AND TRANSITION

Let θ_{max} be the maximum value of θ in the material. The parameters for thermal explosion (criticality) are the maximum temperature excess (θ_{max}), activation energy (β) and Frank-kamanetskii (δ). The point of thermal explosion is identified as the point of bifurcation. The classical treatment by Frank - Kamenetskii of equations (13) – (15) and Table 1 (i. e. $\beta = \alpha = 0$) identified criticality with the marginal disappearance of stationary states. In this case of exponential approximation, $\delta(\theta_{max})$ starts from zero and reaches a maximum and then falls steadily to zero again as θ_{max} increases without limit. The corresponding values of δ_{cr} = 0.8784 (or 2) and $\theta_{max\,cr}$ = 1.1868 (or 1.3778) for slab (or cylinder). In the effect that β is small ($\beta < \beta_{cr}$), $\delta(\theta_{max})$ passes first through a maximum (critical explosion or ignition) then through a minimum (extinction) and finally grows without bound as $\theta_{max} \rightarrow \infty$. The mathematical expression for first criticality (explosion or ignition) is then

$$d\delta/d\theta = 0 \text{ and } d^2\delta/d\theta_{max}^2 < 0. \quad (16)$$

When $\beta = \beta_{cr}$, $\delta(\theta_{max})$ passes through a point of horizontal inflexion corresponding to the unification of the maximum and minimum. This is transition and we call the corresponding values of δ and θ_{max}, δ_{tr} and $\theta_{max\,tr}$. The natural condition for transition is clearly

$$d\delta/d\theta = 0 \text{ and } d^2\delta/d\theta_{max}^2 = 0. \quad (17)$$

If β is further increased beyond β_{tr}, $\delta(\theta_{max})$ grows monotonically with θ_{max}. It is worthy of note that **"The accurate evaluation of β_{tr}, δ_{tr}, $\theta_{max\,tr}$ by the process of guessing a β to evaluate equation (13) over a sufficient range of θ_{max} and revisiting β, would be lengthy"** Boddington et al. (1979). In this context, several articles suffice: Barenblatt et al. (1998), Ayeni (1982 and 1983), Boddington et al. (1983a), Goldfarb and Weber (2006), Ajadi (2011), Britz et al. (2011), Chinyoka and Makinde (2012). Graphical expressions are presented for branched-chain thermal chemistry for combustibles, such as hydrogen-oxygen systems and acetylene-oxygen mixture when $f(\theta) = \theta^m(1+\beta\theta)^n \exp\left(\frac{\theta}{1+\beta\theta}\right)$ (see for example Okoya, 2013a).

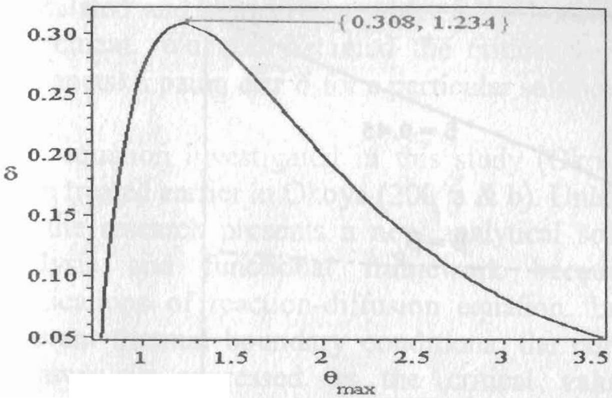

Figure 4: The relation at criticality between the Frank-Kamenetskii parameter δ and the central temperature excess θ_{max} with exponential approximation. The maximum point (1.234, 0.308) corresponds to thermal explosion.

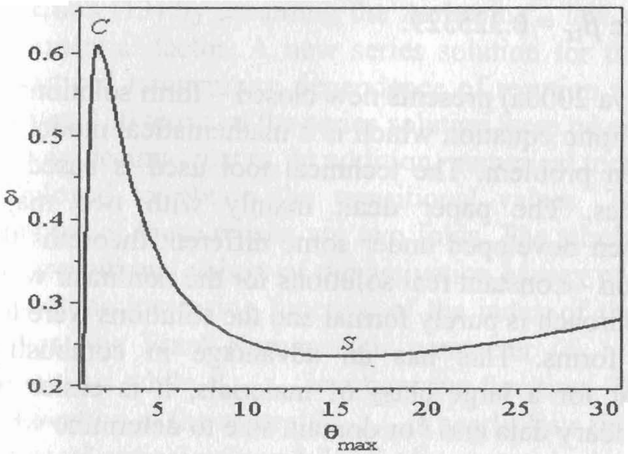

Figure 5: The relation between the central temperature excess θ_{max} and Frank-Kamenetskii parameter δ at criticality for branched chain thermal reaction under sensitized reaction law. The critical point C (1.6525, 0.6085) represents thermal explosion and the point S (15.5525, 0.2336) extinction.

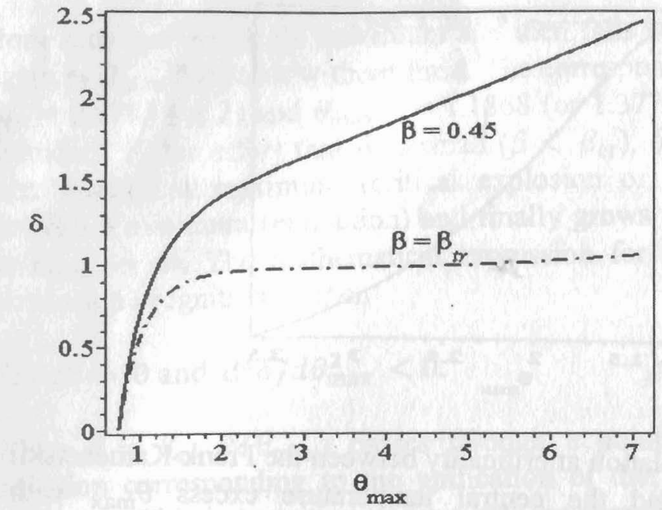

Figure 6: The relation at transition – continuity (loss of criticality or transition) for branched chain thermal reaction under sensitized reaction law. Here $\beta_{\text{tr}} = 0.325329$.

The author (Okoya 2000a) presents new closed – form solutions of a non – linear elliptic equation which is a mathematical model for thermal explosion problem. The technical tool used is based on complex variables. The paper dealt mainly with two major theorems. We then developed under some different theorems the corresponding non – constant real solutions for the nonlinear wave equation. The approach is purely formal and the solutions were left in the general forms. This has an advantage in combustion industries in that for a large class of materials, it is easier for variation of boundary data and / or domain size to determine when a thermal explosion may occur.

The paper represents further work on the preceding paragraph. The author in Okoya (2000b) established four other solutions of a special type of separation of variables in the preceding paragraph. The resulting solutions are presented in terms of trigonometric, hyperbolic and Jacobi elliptic functions. Of particular interest is the Frank-Kamenetskii parameter $\delta \leq 0$. Two theorems were

postulated and comparison with earlier known results is in good agreement. We also obtained the critical value of the Frank – Kanenetskii parameter δ for a particular solution.

The equation investigated in this study (Okoya et al. 2003) has been treated earlier in Okoya (2000a & b). Unlike Okoya (2000a & b), the research presents a new analytical solution via complex analysis and functional framework because of the novel applications of reaction-diffusion equation. Imposing physically relevant thermal boundary conditions, the condition for thermal runaway is expressed as the critical value of the Frank-Kamenetskii parameter. It is worth noting that this research work was done in collaboration with my M.Sc. and Ph.D. students (A. B. Kolawole and S. O. Ajadi, respectively) under the support of University Research Grant no. 11-812-AUY.

Recently, Okoya (2004) studied the one – dimensional slab of equation (13) by assuming the Arrhenius – law with variable pre-exponential factor. A new series solution for the special case of sensitized temperature dependence of reaction rate was obtained. In fact, 120 terms of the series solution were taken for convergence to four decimal places. In addition, numerical techniques were also employed to obtain the transitional values. The most striking features of these results are two folds. The tabular presentation of the transitional values of the activation energy parameter (β_{tr}) is a monotone increasing function of the index of the pre-exponential factor, the Frank-Kamenetskii parameter (δ_{tr}) is with opposite behaviour while $\theta_{\max \text{tr}}$ is a concave function. The second is that the heating at the centre $\theta_{\max \text{tr}}$ is nearly four times as high as the classical critical value while δ_{tr} lies surprisingly close to twice the classical critical value.

More recently, I revisited Okoya (2006a) and analyzed the response of pure diffusion in the kinetic approximation of a combustible mixture and utilized a reduced - order model by Semenov (1959) and Varatharajan and Williams (2000 and 2001) based on the hydrogen - oxygen mixture as well as acetylene -

oxygen systems. Okoya 2013 studied the one –dimensional slab of equation (13), criticality and disappearance of criticality (transition) were investigated for the natural parameters as functions of the emerging parameter. The validity of our numerical procedure is confirmed with tabular comparison of my results against existing one. Graphical illustrations of the critical and transitional values of the natural parameters were presented for various values of the parameters. One was able to understand the structure of branched-chain thermal explosion as compared to the well-known classical thermal explosion theory. The results with suitable approximations for the detailed chemistry may be applied to studying various fuels, mainly hydrocarbons, in air or oxygen mixtures.

As part of my contribution to asymptotic concepts and mathematical methods, direct Taylor series expansion and integration techniques have achieved great success in solving mathematical models of thermal reaction with variable pre – exponential factor as in the special case of sensitized reaction in a slab Okoya (2004). It is worth mentioning that regular perturbation and homotopy perturbation techniques have been successfully used in Okoya (2006c, 2007a, 2008 and earlier papers) respectively for the velocity of higher – order constitutive relations, beyond the Navier – Stokes equations, for stress term using third grade fluid. But the homotopy perturbation and the Taylor series expansion methods requires a large number of terms for accurate simulation.

REACTIVE NON-NEWTONIAN FLUID FLOWS

In contrast to all earlier studies of the fluid systems for thermal explosion, our special type of flows considered here is of higher order than the corresponding Newtonian flow. In fact, for the special flows under consideration, the governing equations reduce to that which one would get for an appropriate generalized Newtonian fluid.

Now, Okoya (2006c, 2007a, 2008, 2011, 2016, 2017), studied the solution set of six simple reactive viscous fluid flows, using the

third grade fluid model, flowing between parallel heated plates, namely, plane Couette flow, plane Poiseuille flow, Generalized Couette flow, flat channel, pipe and cylindrical pipe with surface cooling under realistic conditions. I chose to attack a variety of problems on the basis of my current knowledge and techniques. It is assumed that the flow is laminar, homogeneous and the heat generation term is the generalized Arrhenius form. It is found that the dimensionless temperature solution depends on the non-Newtonian material parameter of the fluid, Λ, viscous heating parameter, Γ, and an exponent, m. Attention is focused on the criticality and disappearance of criticality of the solution set $\{\beta, \delta, \theta_{max}\}$ for various values of Λ, Γ and m, and extensive numerical computations are presented in the form of tables and graphs to show salient features of the solution set.

Furthermore, we have also paid close attention to the one dimensional heat generation and viscous dissipation of the third grade fluid flowing through a cylinder with my young collaborators. Jaiyeoba and Okoya (2012) obtained analytical expressions based on regular perturbation technique involving Reynold's and Vogel's viscosity models. The heat transfer model is also solved numerically and compared favourably well with existing results. The obtained analytical temperature solutions are used to investigate the effects of the heat transfer characteristics which has provided useful and relevant information for handling and processing of coal-base slurries. We have also been able to study criticality and thermal explosion for general spatial dependence of viscosity in a vessel with surface cooling in Ogunseye and Okoya (2017). In this direction, we utilized diagonal Pade approximant, Modified Adomian Decomposition Method and numerical technique. It was also shown in general that the spatial variation of the fluid viscosity does not distort the nature of the transition diagram for both the Newtonian and third-grade fluid flows. Thus, confirming the usefulness of this model in describing the state behaviour.

I wish to emphasize the central importance of the fact that the results of the numerical computation of the articles in the preceding paragraph helped to enhance the understanding of the difference between Newtonian and non-Newtonian thermal explosions.

Furthermore, it is evident that any given reactive third-grade fluid flow of the same mixture under the adiabatic conditions, irrespective of the geometry, exhibit the phenomena of criticality and transition. The overall variation of the triplet at thermal explosion and transitions across all the different geometries and boundary conditions (symmetric and asymmetric heating) is pronounced.

It is advantageous to state that I have other research work yet to be published on annular flow. In the first related work (Okoya 2018a) I numerically investigated the mass and heat transfer in a pressure induced flow of a reactive third-grade fluid with Reynolds' viscosity model through a fixed cylindrical annulus, neglecting material consumption. It was assumed that the walls of the outer cylinder are maintained at the constant temperature T_0 with zero temperature gradient at the inner walls while the non-slip conditions were applied at the cylinder walls. With graphical illustrations, the influence of salient physical parameters on the axial temperature and flow velocity are studied. It was also shown in general that the spatial variation of the fluid viscosity does not distort the nature of the transition diagram for both the Newtonian and third-grade fluid flows. Thus, this is a confirmation of the value of this model in describing the state behaviour.

The second study by the author (2018b) numerically investigates steady, incompressible, exothermic reaction of a third-grade fluid with viscous heating in a cylindrical annulus for both cases of constant viscosity and non-linear viscosity (Reynold's model). The relative rotational motion present between the inner and the outer cylinders induces the flow (i.e. swirling flow between rotating concentric cylinders).

It is interesting to note that, in the last two manuscripts, that I explored new techniques to resolving the problems, taking into account the effect of the gap width on the thermal explosion.

I have done important work on power-law fluids between two-axial cylinders under various thermal boundary conditions at the cylinder walls with application in cylindrical rotary viscometer. In Okoya (2006d and 2007b) I adopted the Barus model for the viscosity. In the first article, I assumed thermal conductivity that is of power law type but with a particular value while the second was generalized thermal conductivity of the same index. Analytical expressions for the critical conditions under hydrodynamic thermal explosion were obtained in Okoya, 2006d as a function of initial temperature and activation energy parameter. On the other hand, transcendental equations for the critical Frank-Kamenetskii parameter were obtained and solved numerically in Okoya, 2007b

NONREATIVE NEWTONIAN FLUID FLOWS

Newtonian's law of viscosity states that shear stress is proportional to the velocity gradient. Fluids that obey this law are known as Newtonian fluids.

Our results in Ayeni and Okoya (1987) are the first on Newtonian fluid. The relevant momentum and energy equations reduced to a nonlinear parabolic equation with appropriate boundary and initial conditions. The influence of pressure dependent viscosity on the temperature of some lubricants was carried out in two main theorems. We demonstrated interplay between the physical fields from which the mathematical problem was formulated.

Furthermore, our work on effect of oscillation in a vertical porous plate with constant suction or injection is found in Ayeni and Okoya (1988). The central equation studied in the research work is a system of coupled non-linear diffusion equations with the hydro dynamical variables (density, velocity & pressure) together with a combustion variable (temperature) in a semi-infinite domain. The authors provided an explicit expression for the effect of oscillation, performed the numerical calculations and displayed graphically the effects of the emerging parameters.

We had also paid close attention to the behaviour of dissipation of boundary layer of certain radial bearing as spelt out in Ayeni et al. (1988a). The resulting partial differential equation of the parabolic type derived from coupled momentum and energy equation with dissipation was analysed for information. We presented rigorous results through the stated theorem and we interpreted the results physically.

Our rigorous analysis of the flow of petroleum oil through pipes in Ayeni et. al. (1988b) revealed, among other things, that under certain hydrodynamic assumptions, our model showed striking difference between steady velocity and temperature profiles of a variable viscosity fluids as well as those of a constant viscosity.

We have also paid attention to the behaviour of blood in the veins as contained in Ayeni et al. (1990). In this paper, we presented numerical and asymptotic solutions for viscous fluid flows in cylinders for a lubricant (blood) by tackling a coupled momentum and energy equations without reactant mass fraction consumption. There was reasonable agreement between the two solutions. Further numerical work revealed that if the length of the cylinder is much longer than its radius, the effect of the slip velocity on the temperature is not important. We also displayed some of the results in tabular form and graphically.

NON-NEWTONIAN FLUID FLOWS

In the effort to find closed form solutions to linear model from Maxwell fluid past an infinite plate with or without the upper surface being free, in collaboration with MSc. Student we studied the flow of a non-Newtonian Maxwell fluid past an infinite plate (Ogunmilade and Okoya (2013)). We employed an analytical approach based on functional framework. Explicit solutions are given in the form of a theorem and the results are compared with

the Newtonian fluid while earlier know analytical solutions were recovered under some limiting conditions.

The theoretical modeling and analysis of a two dimensional steady hydrodynamics laminar flow of an incompressible second grade fluid past a semi-infinite stretching sheet with variable thermal conductivity and viscosity in the presence of heat source/sink was considered with a young collaborator (Akinbobola and Okoya 2015). The basic governing partial differential equations for the dimensionless velocity and temperature are transformed to ordinary differential equations using appropriate similarity variables. The numerical investigation of the variable thermo-physical properties of a second grade fluid over a stretching sheet provides an extension to previous work.

Currently, I have an ongoing project with Dr. Ikenna Ireka which aims to extend for certain unsteady channel flow of a second grade-fluid with power law spatially distributed viscosity results previously developed by Rajagopal in (1982) for constant viscosity. This form of spatial variation in the fluid viscosity may arise from concentration gradient, thermal gradient or boundary induced phase transition in the weakly elastic fluid. Two flow problems are considered in Ireka and Okoya 2018 (i) the Generalized Couette and (ii) the time-periodic plane Poiseuille flows. We proposed a new construct based on the Frobenius method which allows for the derivation of possible closed form solutions to the decoupled transient state equation obtained from the separation of the variables in the flow model. The governing equations for both flow problems are solved numerically using finite volume method. With graphical illustration we explore the influence of the material parameters on the flow.

In vivid contrast, we proceeded in Okoya and Ireka (2018) to seek exact solutions of the partial differential equation arising from the unsteady channel flow of a second grade-fluid for power law spatially distributed viscosity as well as elasticity using standard techniques. Four types of time-dependent flows are investigated in the study. An eigen-function expansion method is used to find the dimensionless velocity distribution. The results however give qualitative description of the velocity and suggesting the possible influence of geometry. This research programme was carried out in collaboration with my former Master's student who eventually earned his Ph.D. abroad.

Adeniyan and Okoya (2018) studied the boundary-layer flow and heat transfer characteristics of an incompressible thermally radiating second-grade fluid on a linearly stretching permeable vertical slender hollow cylinder. The model incorporates the temperature dependent fluid properties as per dynamic viscosity and thermal conductivity. The coupled partial differential equations governing the fluid flow were transformed to nonlinear ordinary differential equations, and then solved numerically using Runge-Kutta-Fehlberg integration scheme. The fluid behavior and heat transfer characteristics within the boundary layer with deformation parameter are analyzed and discussed through simulated tables and codes generated graphs.

Also, Fatunmbi and Okoya (2018) investigated heat transfer process in boundary layer MHD of a viscous dissipative and thermal radiating Micropolar fluid over a stretching sheet in a Darcy porous medium. The fluid material properties such as viscosity and thermal conductivity are assumed to vary with temperature and the cases of prescribed surface temperature and heat flux on the stretching surface are considered. The resulting system of equations from transformation was solved via shooting method alongside fifth-order Runge-Kutta-Fehlberg integration scheme. The key feature of the numerical computation is the influence of physical controlling parameters on the dimensionless temperature which are graphically presented for both thermal conditions on the surface.

From what I have said up to know, it is evident that in order to get physical insight into chemical reactions in different geometries, mathematical modeling has played a prominent role. In the first

instance, it provides physical insight into the safe period when experiments are to be conducted in the laboratory. Secondly, parameter characterization is self-evident from mathematical modeling especially how it affects the thermal explosion and transition with applications in processing industries. Thirdly, mathematical modeling of reactions has made it possible to understand the structure of thermal explosion and branched-chain thermal explosion

It is worthy of note that the cited works in this inaugural lecture contained a more comprehensive and well-documented review of chemical kinetic models in the field of combustion.

FUTURE RESEARCH WORK

There is need to further develop computation tools to handle new models which incorporate more kinetics and/or realistic, industrial–aligned boundary conditions. The intending researcher(s) must note that the calculation of the critical parameters of thermal explosion is not trivial because the temperature solution at the point of explosion is not stable.

Nonaxisymmetric modifications of the discussed problems of thermal explosion in cylindrical vessels need further attention from scientists. These models are tailored to reproduce phenomena observed in combustion engine of vehicles.

The major mathematical models in terms of the considered elementary steps and associated rate of reaction pioneered by Williams (1986) and discussed by Varatherajan and Williams (2000 and 2001) and Okoya (2006a and 2013) for branched chain thermal reactions need further treatment using rules and conditions very close to those considered for thermal reactions. We must numerically evaluate the critical parameters of interest and present instructive properties embedded in the models.

ACADEMIC MENTORSHIP AND MEMORABLE EVENTS

During my National Youth Service year, I was priviledged to handle the tutorial classes in the Department of Mathematics after my redeployment from Sokoto State in late 1983 and since then I had started interacting closely with the university students. Generally, I was very generous with my time and ideas. Hence, my impact on the education and mentorship of the young Nigerian mathematicians and science-based undergraduates since 1984 is unquantifiable. A recount of my welcoming hands was given by one of the then undergraduates in one of the letters I received from a former student which I believe will be appropriate to be shared at this juncture:

"Today was a memorable day for me as I was chanced to see Professor S. S. Okoya by a Divine arrangement at Obafemi Awolowo University, Ife, Nigeria after about 25 years. I feel so fulfilled to know that this wonderful lecturer and old friend has really accomplished so much in his field. He is currently a recipient of the Professorial Chair in Mathematics at University of Lagos, Akoka, Yaba, Lagos, Nigeria, endowed by Pastor Enoch Adejare Adeboye (Baba Adeboye). I met this wonderful man at Great Ife in 1984 and he and his lovely fiancee (was a pleasure to be at their wedding in 1988) practically adopted my friends and I. I still recall many times he opened up his house to us, stocked his kitchen and allowed us to cook, bake and generally relax and have fun. It is indeed a privilege to know you Sir. These are memories time cannot steal away. True respect, Sir."

Mrs Toyin Femi-Ogundiran – nee Dada (aka Big Toyin) Lagos, Nigeria.
Saturday 08/08/2015"

These lines from Big Toyin, as she was popularly called, brought back memories of my living off campus for twenty three years before I moved to reside at the Senior Staff Quarters, OAU in 2007. The comments she made portrays the power in value and importance of human connection and relationships.

Also, I read in one of the letters I received on my birthday celebration from another prominent student and now my friend

who did his B.Sc. project and M.Sc. thesis under my watchful eyes which I think is appropriate:

"To God be the Glory great things He has done. I look back at my life and see how much you have greatly impacted me. You have been a great teacher, a supportive mentor, an admired colleague, an encouraging Father and have been very caring to me. Words cannot express how much you mean to me and my family. As you add a year today, I pray that God will continue to uphold and strengthen you. You have laboured for the growth of many, God will grant you Longevity of life so that you will fully enjoy the fruits of your labour. You are celebrated today and the joy of this day shall remain in your home always in Jesus name, Amen! Happy Birthday Sir."

Dr. Ikenna Ebubechukwu Ireka 20th October 2017 (Germany)
The above remark from Dr. Ireka, my former student, is an indication that lecturers and the students have complimentary roles. The good teacher compliments the good students to have the best world.

I have since kept these testimonials and others (that time and space will not allow) as a reminder to be the best person I can possibly be. The testimonials also remind me that we never know where or when our paths will cross again in life and have motivated me to keep in touch with past students, some of whom are now graduate students in mathematics, financial mathematics, statistics, engineering and other science-based courses, while others have found gainful employment in public and private sectors.

Ladies and gentlemen, it is evident in the reports of my undergraduate and postgraduate students that the inspired feedback is the by-product of one of my goals which was derived from a saying by John C. Maxwell '**we teach what we know, but we reproduce what we are**'.

Being an alumnus, I have a deep passion for proper tutelage of both undergraduate and postgraduate education to produce other great alumni. My students have attested to the fact that they learnt a lot from me through my experience, knowledge, and strategy developed over the years through my special lecture pattern stated below:

1. Define the concept
2. Write things down
3. Explain them to students
4. Connect with earlier studies
5. Illustrate with diagrams where applicable
6. Encourage them to do as many exercises as possible thereby inculcating practice makes perfection
7. Make myself available to students to provide guidance.

I used this methodology for decades and I have evidence of fruits that have abided in the Mathematical community and academic community at large. My guiding principle is based on the saying, "**the mediocre teacher tells, the good teacher explains, but the great teacher demonstrates**". However, my observation in the students I taught in the 1980s and 1990s is that at least one student will ask challenging questions that will keep me on my toes while in this millennium, the story is quite different in that there are no such challenges from the students. I hope there will be improvement on this in the nearest future.

Also, I contributed tremendously to mentorship in Department of Mathematics, OAU and Nigeria at large. Since 1984, I have been an academic father with a listening ear and my doors have always been opened to students and staff till date. I also served as the Patron of National Association of the Mathematical Students of Nigeria (NAMSN) between August 2008 and July 2011 and Staff Adviser from August 2011 to August 2014. I have provided unwavering support (in cash and kind) to the production of the first three volumes of MATHEMA magazine of the NAMSN through my motivation as the Head of Department and the magazine has been produced by NAMSN till date. This was an investment of my time, money, and network which motivated NAMSN in December, 2012 to honour me with an award of excellence during the 50th

Anniversary of OAU in recognition of my selfless and humanitarian contributions to the development of NAMSN OAU chapter and humanity at large.

Moreover, in my quest as the Head of Department to add value to students' knowledge and improve the international outlook of the Department and the University at large, I invited to the department, two eminent mathematicians to provide an interactive forum and give a 3-week tutorials on Real analysis and Algebra. This gesture of bringing foreign scientists to strengthen fields different from my area of specialization is a demonstration of my love for holistic education for Nigerian students. The scientists are Professor Michael Nakamaye from the University of New Mexico, Albuquerque, USA (March 2009 and October 2010) and Professor Vladimir Vershinin from France/Russia (May 2010). The second visit of Professor Nakamaye was in the company of his postgraduate student, Mrs Martha Byrne. They came under the Volunteer Teaching Programme for developing countries strategic group of the International Mathematical Union (IMU) in conjunction with Centre International de Mathematiques Pures et Appliquees (CIMPA) and the U. S. National committee for Mathematics. It has been attested to by the students concerned that the visit under the auspices of International Mathematical Union is a laudable project. In fact, a post graduate student from University of Ibadan also benefitted from the visits and had since visited Professor Nakamaye on a research visit.

Mr Vice Chancellor sir, it will be good to call your attention to the fact that Prof Nakamaye is well known to have good track record in teaching and research. This made the DCSG give him the award to be our guest mathematician lecturer on real analysis.

It is my joy that I work in a profession I truly enjoy and derive satisfaction. One thing I find truly rewarding is supervising postgraduate students and watching them grow in the profession; many of them have become personal friends of mine. My first doctoral graduate from OAU has grown to become a Professor of

Mathematics in this University, Professor Suraju Olusegun Ajadi. I supervised two Ph.D. students at the University of Lagos; the first (Dr. Adetunji Adeniyan) is a senior Lecturer at the University of Lagos while the second (Dr. Ephesus Olusoji Fatunmbi) is a Chief lecturer at Department of Mathematics and Statistics, Federal Polytechnic, Ilaro, Nigeria and many others like, Dr. Ikenna Ebubechukwu Ireka (Germany), Dr. Adebayo Abiodun Aderogba (OAU, Nigeria), Olabode Emmanuel Adeleye (Shell Petroleum Development Company of Nigeria - SPDC), Abiodun Hameed Jimoh (USA), Hammed Ogunseye (South Africa) are scattered all over in industries and academic institutions in Nigeria and abroad. I have published several papers with nearly over twenty collaborators within and outside Nigeria.

In the course of my career in the University, I was invited in 1992 as a Visiting Scientist to the International Centre for Theoretical Physics (ICTP), Trieste, Italy for the first time for a period of four months under Professor James Eells (first Head of Mathematics division 1986 – 1992) and Ms Laurenti who was the Secretary. Professor Eells died in Cambridge on the 14th of February, 2007 but his footprint is unforgettable. The visit funded all expenses except travel cost. This was my first International research visit and I also participated at the International Centre for Mathematical Sciences, Edinburgh, Scotland, UK with full funding through the generous invitation of Professor Sir John Ball FRS (now Director of the Oxford Centre for Nonlinear Partial Differential Equations).

This visit was accompanied with an experience I have shared with many of my postgraduate students and colleagues within and outside this University, but in such an event like this, it is worth repeating: I arrived in ICTP, Trieste, Italy by train through Rome with the then Nigerian Airways from Lagos, Nigeria flight. I was provided with a comfortable office equipped with a desktop computer (which was comparably smaller to the giant main frame computer adapted with punched cards at home) immediately I arrived. Accommodation, library and finance were adequately planned without any stress for me. I met the postdoctoral students

as well as other colleagues. There were two Nigerian Mathematicians on ground at ICTP as at that time, Professors Charles Chidume and G. O. S. Ekhaguere. The conducive atmosphere and interaction at the Centre resulted in two articles within four months. The lesson to learn from this gesture is how well you prepare for your visitor's arrival speaks a lot about who you are and what you can achieve.

My experience as an author during the review process of one of the two articles prepared during my four months visit to ICTP mentioned above is also worth sharing in times like this. This experience could be valuable and encouraging for younger researchers facing the ongoing "Publish or Perish" syndrome and challenges of single authorship. I submitted the research article to the journal of Mathematika, a leading journal of University College London. Four reviewers scrutinized the submitted work and I provided answers to all the issues raised bearing in mind that, then, correspondence was mainly by surface post. I kept my hope alive and the article was eventually accepted and published after two years of scrutiny (Okoya 1994). You can imagine my anxiety as a young researcher (lecturer 1) and the only author waiting on the journal editors' decision for two years. These reviewers' comments, suggestions and correspondence are so valuable to me that I kept them as a reminder of how I started my academic publications. It also built my courage that my training is good enough to chart journals that traversed world-leading publishers such as Elsevier, Springer – Verlag, Taylor and Francis, Pergamon and Hindawi, among others for my research articles.

I am profoundly recognised in the profession, having been lead paper presenter and keynote speaker in many institutions such as at Ladoke Akintola University of Technology - LAUTECH (2014), Obafemi Awolowo University - OAU (2014), Yaba College of Technology - YABATECH (2017) and recently at Land Mark University (2018) etc. during academic programmes and conferences. I have been appointed as external examiner to several institutions in Nigeria and abroad including University of Cape

Town, South Africa, University of Nigeria, Nsukka, Federal University of Technology, Minna, University of Ilorin, Ilorin, Covenant University, Ota, University of Port Harcourt, Port Harcourt, Federal University of Agriculture, Abeokuta, Bowen University, Iwo, Redeemers' University of Nigeria, Ekiti State University, Ado – Ekiti, Ahmadu Bello University, Zaria, Ladoke Akintola University of Technology, Ogbomoso, Olabisi Onabanjo University, Ago - Iwoye. I have also served as external assessors of academic staff for promotion at Universities in Nigeria, Ghana and South Africa.

I was also the treasurer of the Nigerian Mathematical Society (NMS) from 2011 to 2016, Editor – in – Chief, Journal of the NMS (2011 - date) and Editor, Notices of the NMS (2016 - date). I have been a member of several learned societies: Nigerian Mathematical Society (NMS) (1985 to date), Mathematical Association of Nigeria (1988), Indian Academy of Mathematics (2000), American Mathematical Society (2000 to date) and Southern Africa Mathematical Sciences Association (2015 - 2016). As a result, in May 2016, I was honoured with the Fellowship of the Nigerian Mathematical Society (FNMS) as well as the Fellowship of the Mathematical Association of Nigeria (FMAN) in September 2016.

It is also on record that I was a recipient of international Fellowships and grants for visits to institutions in UK, Austria and USA and many times to Italy. Also, twice (1994 in Switzerland and 1998 in Germany) I have been invited to International Congress of Mathematicians to deliver papers. Also, I have participated at conferences and workshops in Africa (Nigeria, Republic of Benin, Namibia and South Africa), Europe (Austria, Germany, Switzerland, United Kingdom and Italy), Asia (Turkey and India) and United States of America. I was Third World Academy of Sciences (TWAS) Associate (1997 – 2000) to Madras, India, Regular Associate to Abdus Salam International Centre for Theoretical Physics, Trieste, Italy (2001 - 2006) and alumnus of Institute of Mathematics and its Application, University of Minnesota, Minneapolis, USA (2010).

Mr Vice – Chancellor Sir, in the pursuit of my career outside OAU, I went on sabbatical leave at Ladoke Akintola University of Technology (LAUTECH), Ogbomoso, Oyo State, Nigeria (1992-1993) and Adekunle Ajasin University, Akungba-Akoko, Ondo State, Nigeria (2004 – 2005). I served in LAUTECH to support my supervisor, Professor R.O. Ayeni who was the pioneer Head of Department of Pure and Applied Mathematics. During my sabbatical leave at Adekunle Ajasin University, Akungba-Akoko, I was appointed the Acting Head, Department of Industrial Mathematics (2004 - 2005). As such, I inherited the problem of missing results of students which I addressed by putting in place a Documentation Committee for handling results and re-organizing the filling system in the Department. Also, I introduced for the first time, a Departmental seminar, where staff can share their research output. It was also during my tenure that the Department was accredited by the National University Commission (NUC) for the Industrial Mathematics programme for the first time.

I also served as an associate staff of the African Regional Centre for Space Science and Technology – English (ARCSSTE-E), located within the OAU campus from its inception in November 1998. I was later appointed as the Postgraduate Coordinator of the centre (2006 - 2009) under the Directorship of Professor Oluwagbenga O. Jegede.

It is on record that Pastor E. A. Adeboye, the General Overseer of the Redeemed Christian Church of God (RCCG) worldwide graduated from the Department of Mathematics, University of Ife (now OAU) in 1967. It is of no surprise then, that his unalloyed passion for Mathematics and the Educational sector at large made the Apapa Family (a group of Churches within RCCG of which he is a member) endow four Professorial Chairs in Mathematics in his name. The Professorial Chairs are domiciled at the following institutions; University of Lagos (2009), University of Ibadan (2010), University of Nigeria, Nsukka (2011), and Obafemi Awolowo University, Ile – Ife (2012). Each of the institutions mentioned above received a fifty million naira donation to institute the Professorial Chair on specific occasions of the Annual Thanksgiving Service of the Apapa Family at the Tafawa Balewa Square, Lagos.

Mr Vice Chancellor sir, in recognition of my research productivity, I was privileged to be the first occupier of the Pastor E. A. Adeboye Endowed Professorial Chair in Mathematics at the University of Lagos (UNILAG), Nigeria for four years (2013-2017). It is worthy of note that, to the best of my knowledge, the procedure for my selection was without any political, ethnic, religious or any other caucus bias, but solely by merit. A board of Trustees was constituted to oversee the Professorial Chair office. During my tenure, I was able to profess mathematics more at the national and international levels through the following:

- Travelled to attend international conferences in Namibia, South Africa and Turkey as well as Annual Conferences of the Nigerian Mathematical Society. The generous fund from the endowment made it possible to foster linkages with colleagues both nationally and internationally in the pursuit of excellence in Mathematics.

- Visited the International Centre for Theoretical Physics (ICTP) Trieste, Italy twice to carry out research activities. Reputable international journals have since published the results of the research visits.

- Delivered two public lectures at the UNILAG Main Auditorium (now Prof. Ade Ajayi Auditorium), and Pastor E. A. Adeboye was at hand to witness the inauguration of the Professorial Chair and the first public lecture - another confirmation of his passion for Mathematics - despite his busy schedule. At the occasion, he gave a brief but impactful address on the Mathematics of Marriage.

- Twelve M.Sc. and two Ph.D. candidates completed their Mathematics postgraduate degrees under my supervision. This feat has been possible only because of the Pastor E. A. Adeboye Professorial Chair.

- A significant effort was expended on the organization and execution of the 1st Pastor E. A. Adeboye Endowed Chair Enrichment Training Workshop in Applied Mathematics, at the University of Lagos (6th – 11th June, 2016) in collaboration with the National Mathematical Centre (NMC), Abuja. Three lecturers from the Abdus Salam International Centre for Theoretical Physics (ICTP), Trieste, Italy as well as two lecturers and four instructors from Nigerian Universities were on hand to train a total of 39 participants. This pioneering workshop was for midcareer students and staff and I drew up an expository curriculum which focused on complex fluid flows and its applications. The participants were diligent students and lecturers willing to enrich themselves in mathematical modelling, analysis and numerics. This workshop in the field of applied mathematics was informative and provided a forum for future collaboration.

- On teaching, after the respected Professor Chike Obi of blessed memory, I happen to be the only Professor of Mathematics to teach differential equation in the Department of Mathematics, University of Lagos. Thereby, revitalising the dying interest of UNILAG students of Mathematics in differential equations.

- In the same spirit of giving back to the mathematics community, I have instituted a Postgraduate course prize tagged the **"First occupier of Pastor E. A. Adeboye Chair/ Professor Samuel Segun Okoya Prize"** in the Department of Mathematics, University of Lagos to be awarded to the Postgraduate student with the best average score in Differential Equations I and II (MAT 813 & MAT 814). The award was listed in 2018 convocation brochure.
In the light of the above achievements through the endowed Professorial Chair in UNILAG, it will not be out of order, if Mathematicians in Nigeria continuously celebrate Pastor E. A. Adeboye for his undying devotion not only to the Mathematics community but also to the Educational sector in general.

Mr Vice Chancellor, Sir, the community service component of my career, spans through local and international levels and finding opportunities for community outreach as documented below.

COMMUNITY SERVICE

It is on record and many can attest to it that during my early days as a lecturer in the Department of Mathematics, I operated an open door policy that welcomed all students offering the departmental large service courses for science and technology. I encouraged those that availed themselves of the friendly atmosphere because I do not put students down if they do not achieve what I think they are supposed to. By this act, I did succeed in transforming the attitudes of students who came in with the mindset that mathematics is very difficult towards finding mathematics interesting.

The Phase B, New mathematics building and furniture funded by Education Task Fund (ETF) 2005/2006 session was handed over to the department by the Physical Planning and Development Unit (PPDU) of the University. This made the movement of the Departmental secretariat to Phase B, New mathematics building on 1st December 2008 under my leadership as Head of Department possible and actualised. The members of Mathematics Department (students and staff) were encouraged by the welfare provision of space and identity provided by the University. The movement from white house to the new Mathematics building (Yellow house) has gone a long way to alleviate the inconveniences of many of our staff in the Department, and the Department has been all the better for this act. In other words, our valuable service to the community is a vital instrument in ensuring that the University's duties and responsibilities are successfully carried out.

Furthermore, under my leadership of the Department between 2008 and 2011, the Department was linked with the International Mathematical Union (IMU) in a project under the Developing Countries Strategy Group (DCSG). DCSG promotes mentoring relationship between seasoned Mathematicians in other countries and African colleagues together with their students as well as long term partnerships. The linkage opened avenue for one of our colleagues (Dr. Praise Adeyemo) who participated in the activities during the first visit of our guest mathematician from the USA in

person of Professor Nakamaye to be invited to USA for a collaborative visit with Professor Nakamaye.

I served the Faculty of Science as the pioneer Chairman of the Dapo Afolabi Most Productive Science Scholar Award Committee between 2010 and 2013 and concluded processing for the selection of the 2010 and 2011 candidates.

As part of my service to the University, I was appointed the Angola Hall Master (2008 -2011). I sustained a physical infrastructural development plan for the University in the hall, ensured a positive image of the hall of residence by working with students leadership organizations in the hall and the University at large, having strategy meetings with Hall Management Committee regularly to move the hall forward and being responsible for the needs of the hall in collaboration with the Dean, Division of Students Affairs

I have also contested for administrative positions in the Univers[redacted] in order to serve the community with my prowess. I contested and was elected as a Congregational representative to the Senate in the 2001/2002 and 2002/2003 academic session and on the floor of Senate I was elected as a member of the Appointments and Promotions Committee for the same period.

A significant service to the community was rendered as the Treasurer and hence a council member of the Nigerian Mathematical Society (NMS) between 2006 and 2011. I encouraged payment of the Reciprocity Annual Dues to the American Mathematical Society (AMS). It is on record that the membership of Nigerian mathematician at AMS grew quickly in a geometric progression during my tenure and this strengthened the linkages between NMS and AMS. I also enjoyed a robust relationship with the executives of AMS and got familiar with other mathematicians in Nigeria and the Diaspora. Afterwards, I was elected by the Council of NMS in 2011 to be the Editor–in–Chief of the Journal of the NMS and Editor for Notices of the Nigerian Mathematical Society in 2016 which had been dormant for four years (I hold these positions till date). As the Editor for Notices, It is worth noting that the Notices of the NMS has been resuscitated after I had previously made clear the importance of keeping the mathematical community in Nigeria and the Diaspora informed of our social and academic activities. Also, the unique opportunity of hard copies had made it possible to widely circulate the Notices among mathematicians at our meetings across the nation.

Also during my tenure as the Editor- in – Chief of JNMS, the Journal has been able to prove its worthiness of international acceptability especially when she could live up to being hosted at the reputable Elsevier Publishing House (2015, volume 34 issues 1-3 and 2016, volume 35 issue 1) through the Memorandum of Understanding (MoU) with financial commitment involving the Committee of Vice Chancellors and TetFund. This improved the international outlook of the Journal of the NMS (JNMS) with large numbers of international and Nigeria-based authors publishing therein.

However, the Elsevier Publishing house had to shut down on us due to the missing link between the Committee of Vice Chancellors, TetFund, the Federal Ministry of Education and the Elsevier Publishing House. Mr Vice Chancellor sir, on behalf of the Mathematics community, I want to give a clarion call to all concerned that such opportunity to our researchers should always be cherished and protected to give our research output, Nigerian journals and Institution a great reputation in the international community. Afterall, the contents of our journal actually merited publication in Elsevier Publishing house but for continuous funding support from the Nigerian government. A good product that is not adequately advertised may not bring sufficient profit. The council of NMS is hopeful that the missing link between the parties would be found so that we can be restored back to the Elsevier system.

Nonetheless, my personal search for a way to see that the journal of NMS sustains its international outlook made me enter into a MoU with the Abdul Salam ICTP in 2016 on behalf of the Society. This MoU signed during one of my visits to the centre gave JNMS access to the Open Journal System (OJS) of the centre for wider accessibility and to promote its image.

I am happy to inform you that new facets of the Notices have evolved and I have published three issues till date and the journal is visible at the OJS of Abdul Salam ICTP. This has recently attracted the London Mathematical society who requested to be informed of our activities. The implication of this is that the NMS under my tenure as treasurer, Editor of Notices and Editor-in-Chief of JNMS has gained feasibility and acceptability, in Nigeria, Europe, the UK and the USA.

In my quest to add value to the mathematics community and encourage hard work among the students in the Department of Mathematics, Obafemi Awolowo University, I instituted a prize called **"The Professor Samuel Segun Okoya Prize"**, to be awarded to the best graduating student in MTH 417 – Fluid Dynamics I on the condition that the class of Degree is not less than the Second Class Upper Division (2:1) with a minimum score of 60%. The award had since commenced in 2011/2012 academic session and the previous recipients are listed below:

YEKEEN, Abiola Olatunde , 2011/2012 session
OLASUPO, Christopher Olaoluwa, 2012/2013 session
OGALA, Wilson Ifeanyi, 2013/2014 session
OLUSANYA Oluwafunmilola, 2014/2015 session
DUROTOYE Silas Adeyinka, 2015/2016 session
KOLAWOLE, Rasaq Olumide, 2016/2017 session

Community service continues as I laboured in the vineyard of God. As a young convert at the Full Gospel Business Men's Fellowship International, Nigeria, Ile – Ife Main chapter in 1989, I needed discipleship and found myself at the Redeemed Christian Church of God (RCCG) located behind Urban Day Grammar School through a 'guardian angel' that ministered early to me in Lagos. I served at RCCG as Pastor –in – Charge (1996 - 2000) and was ordained an Assistant Pastor by Daddy G. O. Adeboye in 1997. I started a newsletter about the Bible as the Parish Pastor and this metamorphosized into a Christian book titled "How it should be". This book was co-authored with my wife (Deaconess). Also, we were able to build the house of God through the support of people (far and near without levying) under my leadership despite the communal crises. There and then, we carried out this special calling at The Amazing Grace of God Evangelistic Ministry (The Amazing Grace of God Students' Association - TAGGSA and other arms) and in 2010 my family moved to New Covenant Church, Agodi GRA Centre, Ibadan (a special home that God has prepared for us) under the General Overseer, Rev. (Dr.) Paul Jinadu. I love this family of God.

Mr Vice Chancellor sir, this inaugural lecture will not be complete without giving some recommendations.

RECOMMENDATIONS

As exemplified in this lecture, it will be appropriate for researchers and lecturers in their different fields could relate their study area to the concepts of mathematics already learnt from Mathematicians as Mathematics is best handled by mathematicians. However, if there are gaps in the contents of our curriculum to fill, members of staff in Science, Technology, Engineering and Mathematics (STEM) of this University and Universities in Nigeria at large can form a think- tank (group) to fashion out a synergy of ideas in applicable mathematics which will be taught by mathematicians. After all, there has been feed - back from some of our students on postgraduate studies outside the country for courses such as Mechanical, Electrical - Electronics, Computer Engineering, and Robotics, having offered Mathematical Methods (MTH 201 and MTH 202) from Mathematics Department OAU. In fact some of them claim it was then, they knew the relevance of vector analysis, matrices and tensor analysis that they were taught in their undergraduate days.

Also, some of my research output discussed in this lecture confirmed some experimental results of combustion. One of such is when there is incomplete combustion in the cylinder of the engine of the vehicles and generators; one of the waste products released is carbonmonoxide (CO). Air pollutant (CO) increases the risk of symptoms such as irritation to the eyes, nose and throat, as well as upper respiratory infections such as bronchitis and pneumonia. Therefore there is need for regular maintenance of our cars, other vehicles and generators or removal of smoky vehicles on our highways and worn out generators from our backyard to ensure that the air quality in Nigeria meets the standard of World Health Organization. This might have been one of the reasons for introducing "Road Worthiness Certificate" for vehicles but unfortunately the issuance and execution procedure cannot achieve the goals. There is therefore the need to review the issuance and execution procedure.

From the vantage point of a mathematician actively researching the combustion and combustibility of materials, mathematical modelling has helped prepare grounds for the organization and fabrication of prototypes, provide solutions and relevant explanations to complex physical system occurring in nature and helped to provide relevant predictions that informs us on protective and preventive measures to the devastating effects resulting from combustion in our planet. In this regard, I recommend that this relevant (mathematical) tool be properly harnessed in the process of strategic planning towards national development.

Mr Vice Chancellor Sir, and my distinguished audience, most likely you entered this hall with the notion that explosion is all about misfortune, but from the mathematical modelling point of view we can all agree that there are many "faces" of thermal explosion.

At this junction, in our University that is known for learning and culture, it is cultural to recognize and appreciate selected people that our paths have closed in the course of my journey.

CLOSING REMARKS

Mr. Vice Chancellor, Sir, distinguished audience, my engagement in the reported research work and visits abroad as mentioned in this inaugural lecture, were not aimed at achieving success, but to find scientific truth. However, I have found both. In fact, it is evident that reward without responsibilities is indulgence and not fulfilling. If not for my creator through our Lord Jesus Christ, where would I have been, what would I have become and what would have been the report of my stewardship. In about 60 years, I have tasted and found that the Lord is good, merciful and faithful and I have really enjoyed God's favour in my life and family. Consequently, "I thank You and praise You, O God of my fathers; You have given me wisdom and might, And have made known to me what I asked of You, ..." Daniel 2: 23. Glory be to your name.

In addition, there are so many people that I cannot possibly list them all without inadvertently leaving out someone important whose paths have crossed mine and who encouraged, inspired and contributed to my journey in life. As such, I will just list representatives of such. I thank Mr Vice Chancellor, Professor Eyitope Ogunbodede and all his predecessors (too numerous to mention) that have granted me the conductive atmosphere to seek truth as part of my academic responsibilities and nothing but the truth, the permission to attend conferences and participate in research activities in and outside this University, as well as the opportunity to serve my department. This feat would not have been possible without the cooperation of former Deans of the Faculty of Science and heads of Department of Mathematics for their foresight and support. I really appreciate their gesture while in office for my career development. The contributions of my colleagues who stood in the gap during my trips abroad are well appreciated.

Also I want to thank the University Management under the leadership of the then Vice – Chancellor, Professor Michael Faborode for actualizing the dream of Mathematics Department, during his tenure, to occupy the new Phase B of Mathematics Building between 2008 and 2011 when I was the head of

department. This kind gesture had made all academic, technical and administrative staff to be together under the same roof. The efforts of all former Heads of Mathematics Department who had been willing to achieve this goal are gratefully acknowledged.

My special thanks goes to Pastor E. A. Adeboye for joyfully receiving me during the official presentation (led by the Chairman, Board of Trustees (BoT), Professor Babajide Alo) in his residence at the Redemption Camp, Km 45, Lagos – Ibadan, Express way, Ogun State in February 2014 as the first occupier of Pastor E. A. Adeboye Endowed Professorial Chair in Mathematics, University of Lagos, Lagos. Also, I appreciate Daddy G. O. for his presence to witness the inauguration of the Professorial Chair and first Annual Lecture in February 2015 at the Unilag Main Auditorium (now Ade Ajayi Auditorium), despite his busy schedule. Professor Oluwatoyin Ogundipe, the current Vice Chancellor of UNILAG and an Alumnus of Great Ife is also appreciated for being part of the team who presented me to Daddy G. O. and on the other hand representing the then Vice – Chancellor, Professor Rahamon Bello, and presiding over the 2nd and valedictory lecture. This arrangement would not have been possible without the input of the Vice Chairman of the BoT and leader of the representative of RCCG pastors of Apapa parish in the BoT, Pastor Idowu Iluyomade. You are sincerely appreciated.

In like manner, I express my gratitude for the opportunity to serve the mathematics community and the University of Lagos at large during my four year appointment as the Pastor Enoch Adeboye Outstanding Professor of Mathematics at the University of Lagos, Yaba, Lagos. I sincerely appreciate the former Vice – Chancellor (UNILAG), Professor Rahamon Bello for the numerous approvals he made based on the recommendations of the diligent BoT. The Board of Trustees was incredibly supportive through the many hurdles of the office, as the first occupier of this prestigious Chair. Needless to say, I did enjoy the rich and energetic atmosphere that the BoT fostered and will forever cherish the relationships built over the last 4 years. I appreciate the former head of Mathematics department at UNILAG where the Professorial Chair was domiciled, in the person of Professor Ray Okafor (2009 – 2014) for laying good examples, and providing guidance and support. In fact, he was at hand during my early days as the first occupier while his robust and motivating reports on me to the BoT were commendable. I sincerely appreciate the contributions of the immediate past Head of Mathematics Department Professor S. O. Ajala (2014 – 2017) for also being my eyes at the BoT meetings. I must say that I am extremely fortunate to have contact with these people and I am deeply appreciative of their efforts. In the same vein, I extend my thanks to the colleagues and staff members (Teaching and non-teaching) in the Department of Mathematics, UNILAG for making my stay memorably.

I am indebted to my M.Sc. and Ph.D. supervisor in the Department of Mathematics, OAU in the person of Late Professor R. O. Ayeni. He taught me hard work and encouraged me to be selfless in tutoring undergraduate students while in training. I recognise and appreciate Professor E. A Akinrelere who introduced me to vectors and mechanics in the late 70's as a fresher, and later taught me vectorial mechanics and fluid dynamics at the undergraduate and postgraduate levels. The discipline and focus that he instilled in me made me to love applicable mathematics. It is on record that my class of M.Sc. set was the second to take Applied Mathematics courses in the Department of Mathematics, OAU in 1984/85 academic session and both lecturers were at hand to nurture and supervise the students. Thank you for your labour of love.

I gratefully acknowledge the kind hospitality of Abdus Salam ICTP, Trieste, Italy for the opportunities given to me on many occasions to visit the Centre. The visits had greatly enhanced my research output and international recognition.

Special thanks to friend and colleague Dr. Ehimika Clement Onime of the Abdus Salam ICTP Trieste, Italy for his IT support and various other contributions during and after my visits to the centre. Special thanks to Professor Charles Chidume, a retiree as Researcher at Mathematics section and Postgraduate Diploma

Coordinator at the Centre for his support and encouragement before and during my research visits. `I want to thank Professor G. O. C. Ekhaguere for his patience as he served as a useful guide when I first started learning LaTex software at ICTP in 1992.

It is on record that my subsequent international activity was strongly enhanced after my first visit to the International Centre for Mathematical Sciences (ICMS), University of Edinburgh, Scotland, UK, for a research visit and to participate in a conference where I met the distinguished Professor, Sir John Ball. I am grateful for his encouragement and labour of love to get me to Europe for the first time with the support of Professor James Eells of ICTP. Professor John Ball eventually became the President of the IMU and I enjoyed numerous priviledges then and thereafter.

I would like to acknowledge the contribution of Dr. Ikenna E. Ireka and Pharmacist Funmbi Okoya for proofreading the first draft of this Inaugural lecture. Also, my thanks go to Mrs 'Wunmi Aluko for proofreading the final draft of this Inaugural lecture.
The efforts of the sponsors and the members of each of the committee involved in the execution of this inaugural lecture is highly appreciated and put on the record of good performance.

My utmost gratitude to the pastorate of the New Covenant Church, Agodi Centre, Queen Elizabeth road, Agodi G.R.A., Ibadan under the auspices of Rev. Olufemi Oyelade who is also the Deputy General Overseer of the New Covenant Church, World Wide except the U.K. and U.S.A., and his wife, Rev. (Mrs) Oluyemisi Oyelade and the Assistant Pastor Rev. Olutoye and his wife, Dr. (Mrs) Olufunmilayo Oyero for their spiritual guidance, true love, care and prayers. I thank God for the concerns and supports of all the members of the pastoral team and church workers as well as the congregation. I am grateful to Rev. 'Femi and Rev. Professor (Mrs) 'Funmi Soetan, the Conference Pastor of the New Covenant Church, Ife City Centre, Ile - Ife, as well as pastoral team, church workers and the congregation for their help and spiritual nourishments on my children and for welcoming my wife and I on some occasions.

How can I do justice to my success story without mentioning these family connections. My gratitude to God for the lives of my late parents (Mr. Jackson Olatunji Ishola and Mrs. Felicia Aina Okoya – nee Osho) as well as my late mother – in – law (Mrs Comfort Olawunmi Olubakin) and late father - in – law (Mr Alfred Adetunji Olubakin) for the gifts / seeds of life God blessed them with and for the Okoya and Olubakin family relations. I did enjoy financial supports from my parents' family connections at the secondary and university levels for which I am grateful.

Ladies and gentlemen, I have lived the life of an academic and a research mathematician, and I feel I have been most fortunate in life. I have a wonderful family; my darling wife and mother of my children, Dr. (Mrs.) Aderonke Adetutu Okoya who is an environmental Chemist, a colleague, and a co-worker in this University. Her intelligence, care and the grace of God upon her life in the conduct of the home affairs during my research visits across the globe from 1992 to date are unquantifiable. In fact, it gives me the opportunity to concentrate in the race that led to my being a leading figure in the international community of applied mathematicians. For this I say thank you, my sweet heart, soul mate, my pillar of support, my guardian angel and my only wife home and abroad, Dr (Mrs) A. A. Okoya. Also, my darling wife is always confident in her disposition, she contributes meaningfully during discussions at home and support me to proofread my letters and articles because I identified her painstaking and meticulous attitude to academic issues. She doesn't hesitate to ask questions, as well as being assertive when necessary. I salute my wife for facing the struggles of balancing family and career.

I have two sons and one daughter who are alumni of this great citadel of learning - OAU. My first son, Timileyin Okoya and daughter, Dr. IfeOluwa Babayomi are unavoidably absent today but are well represented by their brother, the baby of the house, Pharmacist Funmbi Okoya at this occasion. They are all doing well in their chosen professions and that I am proud of. Thank you all

for making me proud. I also wish to register my appreciation to my son-in-law, Dr Opeyemi Babayomi, who has always been a son to me, my granddaughter, Oluwatishe Babayomi and all the members of Babayomi family. I love you all.

Mr Vice Chancellor, Sir, Distinguished Ladies and Gentlemen, My 34 years in service has taught me to love people, work hard, study, develop and share my expertise in my discipline, some people shared my view while others present their own views which I also learnt from. Afterall "W" when turned upside down is "M", when viewed from the right side is "3" and when viewed from the left side it is "E". The fact that I do not see things in the same perspective with you in your own field does not make me wrong. A word can mean different things to different people.

Thank you for your patience and attention.

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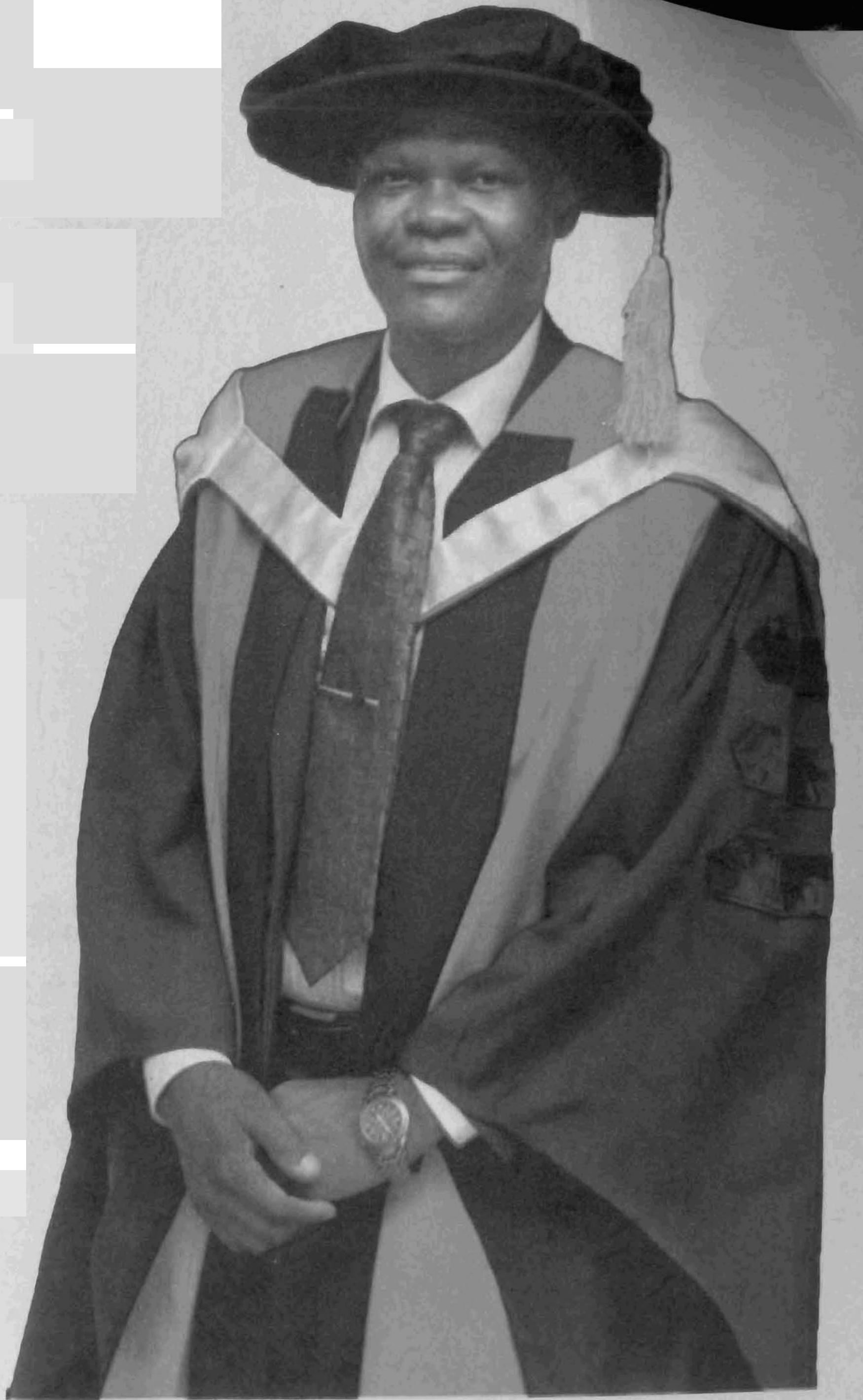

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