NESTED BALANCED INCOMPLETE BLOCK DESIGNS OF THE SERIES I AND II TYPE

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ABSTRACT

Suppose there exists a Balanced Incomplete Block (BIB) design (v, b, r, k, λ) for which an initial block solution could be obtained from t-initial blocks. Equally if it is possible to divide each initial block into m sub-blocks from the initial block for generating a BIB design with v treatments arranged into blocks of size k_2. Then by systematic development of these initial blocks, we have NBIB designs for both the series II and I. Designs constructed in this paper, which are referred to as of series-I and series-II, are indeed of the Nested Balanced Incomplete Block Designs (NBIBD's) type and they do exist for some values of v. Meanwhile, NBIBD's for series-I exist for all odd values of v, while NBIBD's for series-II equally exist for all even values of v. Also if sequence of the treatment (v) for series-II is one fewer than series-I, both series-I and series-II give the same initial block and consequently the main block of size k_1 for series-I is equal to k_1 for series-II, and correspondingly the size of the sub- blocks, k_2 for series-I is equal to k_2 for series-II. Other design parameters exhibit similar interesting symmetry.

Keywords: Nested Design, Incomplete Block, Balanced Incomplete Block, Series-I, Series-II,

1. INTRODUCTION

Heterogeneity in the experimental material is a critical problem to be reckoned with in the statistical design of scientific experiments; infact, if it is not suitably taken care of in designing of an experiment it is likely to over shadow the real treatment differences making them undetected, unless they are large enough Rajender, (19******). Occasionally, one can find a nuisance factor, which, though not of interest to the experimenter, may contribute significantly to the variability in the experimental material. Various levels of these factors are used for blocking. It should be noted that, blocking is the technique used to bring about homogeneity of experimental units within a sub-division of the experimental material, so that the treatment contrasts are estimated, making use of the intra-block information, with higher efficiency. For the experimental situations where there is only one nuisance factor, the block designs are used.

When two such cross – classified factors are present row – column designs such as Latin Square, lattice square, youden square, generalized youden, pseudo youden designs etc. are being used. In many fields and laboratory experiments the experimental units or conditions differ due to several factors, which influence the response under study. It might not always be possible to remove such heterogeneity in response due to the factors other than treatments by blocking alone. There are experimental situations in which there are one or more factors nested within the blocking factor. This shall be illustrated in what follows in section 2.

2. NESTED DESIGNS

In nested designs the treatments are formed with different levels of factors, where one factor is nested within the other in the experiment. Again in some experiments the levels of one factor (e.g. Factor B) are similar but not identical for different levels of another factor (e.g. Factor A). This arrangement is called nested design or hierarchical design.

For example, suppose a company buys raw material from three different suppliers. The company may want to know if raw materials purity depends on the supplier. Four lots of raw materials are selected at random from each of the suppliers. Three different measures of purity are made on each batch of raw material.

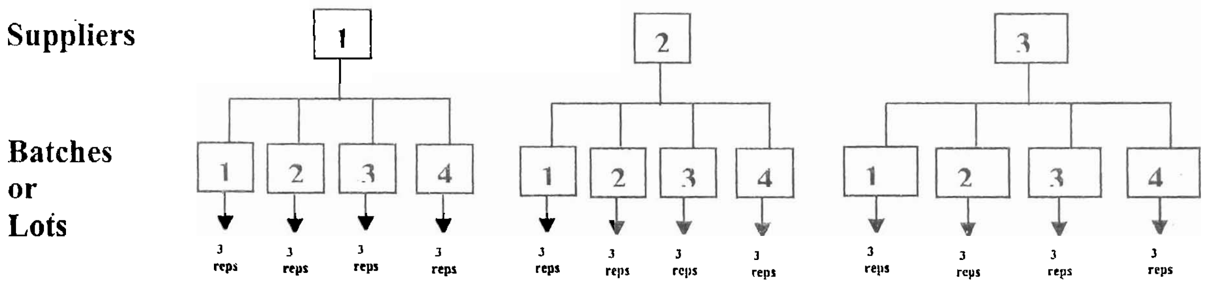

Figure 1: flowchart of nested design on raw material supplier.

The lots (batches) of raw materials are nested within suppliers. Lot 1 under supplier 1 is not the same lot as lot 1 under supplier 2 or 3. Another diagram that captures adequately the foregoing statement is shown in figure 2 below.

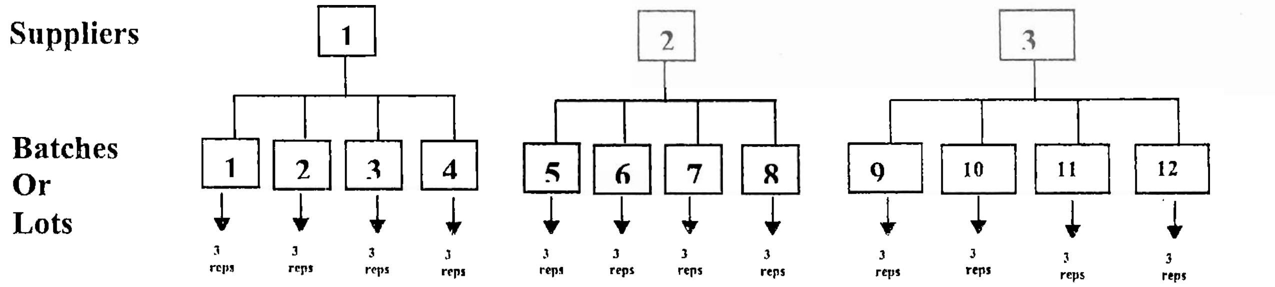

Figure 2.2: Another version of the flow chart in figure 1

Figure 2 shows more clearly that each lot is dependent on supplier, that is, raw material' lots are nested within suppliers. Suppose the lots are children and the suppliers are parents, each lot is uniquely tied to its supplier just as a child is to its parents.

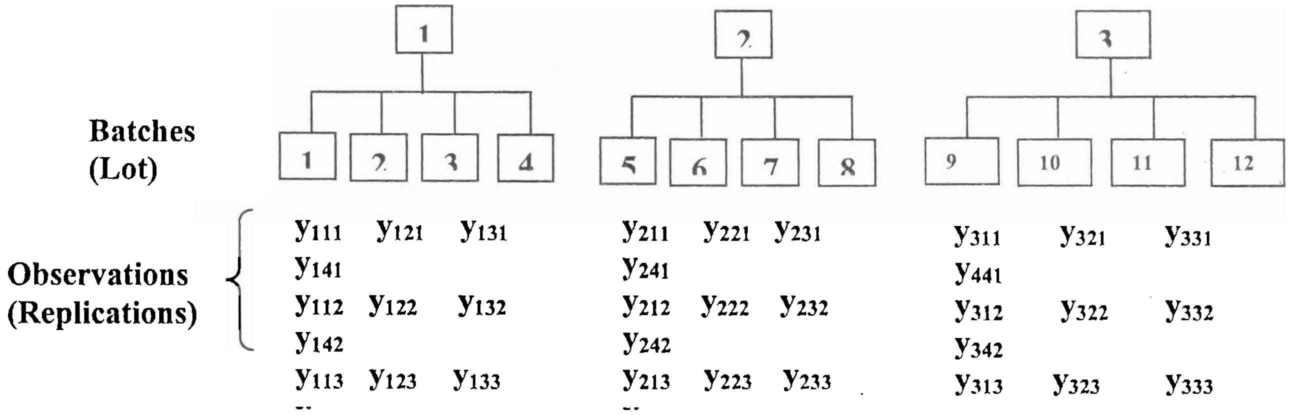

Figure 3: Data structure for the two-stage nested design.

3. METHODOLOGY

Suppose there exists a BIB design (v, b, r, k, λ) for which an initial block solution based on t-initial blocks is available. Equally if it is possible to divide each initial block into m sub-blocks from the initial block for generating a BIB design with v treatments and block size K_2. Then clearly by developing these initial blocks, we get a NBIB design with parameters

$$v = v^1,\ r = r^1,\ b_1 = b^1,\ k_1 = k^1,\ \lambda_1 = \lambda^1,\ b_2 = mtv^1,\ k_2,\ \lambda_2 = r^1\,(K-m)/m\,(v-1) \qquad (3.1)$$

The following models for the series of NBIBD type can always be constructed:

Series-1: $v = 2t+1 = b_1 = 2t+1,\ b_2 = t\,(2t+1),\ k_1 = 2t,\ k_2 = 2,\ r = 2t,\ \lambda_1 = 2t-1,\ \lambda_2 = 1$

Series-11: $v = 2t,\ b_1 = 2t-1,\ b_2 = t\,(2t-1),\ k_1 = 2t,\ k_2 = 2,\ r = (2t-1),\ \lambda_1 = 2t-1,\ \lambda_2 = 1$

Thus, NBIBD for Series-1 is obtained by developing the initial block using

[(1, v-1), (2, v-2), ... (t, v-t)] mod 2t+1

(3.2)

Similarly,

NBIBD for Series-11 is equally obtained by developing the initial block using

[(1, v), (2, v-1), ..., (t, v-t+1)] mod 2t-1 by taking v^{th} treatment as invariant

(3.3) Rajender and Gupta (1993).

4. CONSTRUCTION OF NBIBD FOR SERIES-I

NBIBD for Series-I using the expression 3.1 for v=13, t=6 is cyclically developed to arrive at the solution below.

[(1,12),	(2,11),	(3,10),	(4,9),	(5,8),	(6,7)]
[(2,13),	(3,12),	(4,11),	(5,10),	(6,9),	(7,8)]
[(3,1),	(4,13),	(5,12),	(6,11),	(7,10),	(8,9)]
[(4,2),	(5,1),	(6,12),	(7,12),	(8,11),	(9,10)]
[(5,3),	(6,2),	(7,13),	(8,13),	(9,12),	(10,11)]
[(6,4),	(7,3),	(8,1),	(9,1),	(10,13),	(11,12)]
[(7,5),	(8,4),	(9,2),	(10,2),	(11,1),	(12,13)]
[(8,6),	(9,5),	(10,3),	(11,3),	(12,2),	(13,1)]
[(9 7),	(10,6),	(11,4),	(12,4),	(13,3),	(1,2)]
[(10,8),	(11,7),	(12,5),	(13,5),	(1,4),	(2,3)]
[(11,9),	(12,8),	(13,6),	(1,6),	(2,5),	(3,4)]
[(12,10),	(13,9),	(1,7),	(2,7),	(3,6),	(4,5)]
[(13,11),	(1,10),	(2,8),	(3,8),	(4,7),	(5,6)]

The parameters of the design above are specified as;

$V = 13$, $r = 12$, $b_1 = 13$, $k_1 = 12$, $\lambda_1 = 11$,
$b_2 = 78$, $k_2 = 2$, $\lambda_2 = 1$,

5. CONSTRUCTION OF NBIBD FOR SERIES-II

NBIBD for Series-II using Expression 3.2 for t-initial block t=7; v=14 is developed cyclically to arrive at the solution below.

[(1,14),	(2,13),	(3,12),	(4,11),	(5,10),	(6,9),	(7,8),]
[(2,14),	(3,1),	(4,13),	(5,12),	(6,11),	(7,10),	(8,9),]
[(3,14),	(4,2),	(5,1),	(6,13),	(7,12),	(8,11),	(9,10),]
[(4,14),	(5,3),	(6,2),	(7,1),	(8,13),	(9,12),	(10,11),]
[(5,14),	(6,4),	(7,3),	(8,2),	(9,1),	(10,13),	(11,12),]
[(6,14),	(7,5),	(8,4),	(9,3),	(10,2),	(11,1),	(12,13),]
[(7,14),	(8,6),	(9,5),	(10,4),	(11,3),	(12,2),	(13,1),]
[(8,14),	(9,7),	(10,6),	(11,5),	(12,4),	(13,3),	(1,2),]
[(9,14),	(10,8),	(11,7),	(12,6),	(13,5),	(1,4),	(2,3),]
[(10,14),	(11,9),	(12,8),	(13,7),	(1,6),	(2,5),	(3,4),]
[(11,14),	(12,10),	(13,9),	(1,8),	(2,7),	(3,6),	(4,5),]
[(12,14),	(13,11),	(1,10),	(2,9),	(3,8),	(4,7),	(5,6),]
[(13,14),	(1,11),	(2,11),	(3,11),	(4,11),	(5,11),	(6,7),]

The parameters of the design above are specified as;
$v = 14$, $r = 13$, $b_1 = 13$, $k_1 = 14$, $\lambda_1 = 13$,
$b_2 = 91$, $k_2 = 2$, $\lambda_2 = 1$,

TABLE 1: Detailed Table on Nested Balanced Incomplete Block Designs, $6 \leq v \leq 30$, $5 \leq r \leq 29$

S/N	v	b_1	b_2	r	k_1	k_2	Λ_1	λ_2		Initial Blocks
1.	6	5	15	5	6	2	5	1	S_2	[(1,6), (2,5), (3,4)]
2.	7	7	21	6	6	2	5	1	S_1	[(1,6), (2,5), (3,4)]
3.	8	7	28	7	8	2	7	1	S_2	[(1,8), (2,7), (3,6), (4,5)]
4.	9	9	36	8	8	2	7	1	S_1	[(1,8), (2,7), (3,6), (4,5)]
5.	10	9	45	9	10	2	9	1	S_2	[(1,10), (2,9), (3,8), (4,7), (5,6)]
6.	11	11	55	10	10	2	9	1	S_1	[(1,10), (2,9), (3,8), (4,7), (5,6)]
7.	12	11	66	11	12	2	11	1	S_2	[(1,12), (2,11), (3,10), (4,9), (5,8), (6,7)]
8.	13	13	78	12	12	2	11	1	S_1	[(1,12), (2,11), (3,10), (4,9), (5,8), (6,7)]
9.	14	13	91	13	14	2	13	1	S_2	[(1,14), (2,13), (3,12), (4,11), (5,10), (6,9),

										(7,8)]
10.	15	15	105	14	14	2	13	1	S_1	[(1,14), (2,13), (3,12), (4,11), (5,10), (6,9), (7,8)]
11.	16	15	120	15	16	2	15	1	S_2	[(1,16), (2,15), (3,14), (4,13), (5,12), (6,11), (7,10), (8,9)]
12.	17	17	136	16	16	2	15	1	S_1	[(1,16), (2,15), (3,14), (4,13), (5,12), (6,11), (7,10), (8,9)]
13.	18	17	153	17	18	2	17	1	S_2	[(1,18), (2,17), (3,16), (4,15), (5,14), (6,13), (7,12), (8,11), (9,10)]
14.	19	19	17	18	18	2	17	1	S_1	[(1,18), (2,17), (3,16), (4,15), (5,14), (6,13), (7,12), (8,11), (9,10)]
15.	20	19	190	19	20	2	19	1	S_2	[(1,20), (2,19), (3,18), (4,17), (5,16), (6,15), (7,14), (8,13), (9,12), (10,11)]
16.	21	21	210	20	20	2	19	1	S_1	[(1,20), (2,19), (3,18), (4,17), (5,16), (6,15), (7,14), (8,13), (9,12), (10,11)]
17.	22	21	231	21	22	2	21	1	S_2	[(1,22), (2,21), (3,20), (4,19), (5,18), (6,17), (7,16), (8,15), (9,14), (10,13), (11,12)]
18.	23	23	253	22	22	2	21	1	S_1	[(1,22), (2,21), (3,20), (4,19), (5,18), (6,17), (7,16), (8,15), (9,14), (10,13), (11,12)]
19.	24	23	276	23	24	2	23	1	S_2	[(1,24), (2,23), (3,22), (4,21), (5,20), (6,19), (7,18), (8,17), (9,16), (10,15), (11,14), (12,13),]
20.	25	25	300	24	24	2	23	1	S_1	[(1,24), (2,23), (3,22), (4,21), (5,20), (6,19), (7,18), (8,17), (9,16), (10,15), (11,14), (12,13),]
21.	26	25	325	25	26	2	25	1	S_2	[(1,26), (2,25), (3,24), (4,23), (5,22), (6,21), (7,20), (8,19), (9,18), (10,17), (11,16), (12,15), (13,14)]
22.	27	27	351	26	26	2	25	1	S_1	[(1,26), (2,25), (3,24), (4,23), (5,22), (6,21), (7,20), (8,19), (9,18), (10,17), (11,16), (12,15), (13,14)]
23.	28	27	378	27	28	2	27	1	S_2	[(1,28), (2,27), (3,26), (4,25), (5,24), (6,23), (7,22), (8,21), (9,20), (10,19), (11,18), (12,17), (13,16), (14,15)]
24.	29	29	406	28	28	2	27	1	S_1	[(1,28), (2,27), (3,26), (4,25), (5,24), (6,23), (7,22), (8,21), (9,20), (10,19), (11,18), (12,17), (13,16), (14,15)]
25.	30	29	435	29	30	2	29	1	S_2	[(1,30), (2,29), (3,28), (4,27), (5,26), (6,25), (7,24), (8,23), (9,22), (10,21), (11,20), (12,19), (13,18), (14,17), (15,16)]

Key: Round brackets ( ) are used for blocks of size k_2 (sub-blocks) while square brackets [ ] are used for blocks of size k_1 (main blocks). Also s_1 represent series-I while s_2 represent series-II

6. CONCLUSIONS

NBIBD's for series-I exist for all odd values of v, while NBIBD's for series-II equally exist for all even values of v. Also if sequence of the treatment (v) for Series-II is one fewer than Series-I, it is observed that both Series give the same initial blocks and consequently k_1 for Series-I is equal to k_1 for Series-II, k_2 for Series-I is equal to k_2 for Series-II, λ_1 for Series-I is equal to λ_1 for Series-II and λ_2 for Series-I is equal to λ_2 for Series-II.

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