

# GENERALIZED PARTIAL CREDIT AND GRADED RESPONSE MODELS IN POLYTOMOUS MATHEMATICS ACHIEVEMENT TEST ITEMS IN ABILITY ESTIMATION IN LAGOS STATE

BY

# IWINTOLU RUKAYAT OYEBOLA (EDP12/13/H/0231)

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Prof. E. R. I. Afolabi	Date
Supervisor	
Dr. (Mrs.) S. A. Ehindero	Date
Acting Head of Department	



### **DEDICATION**





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#### LIST OF ABBREVIATIONS AND ACRONYMS

**Acronyms** Title

1PLM One Parameter Logistic Model

2PLM Two Parameter Logistic Model

3PLM Three Parameter Logistic Model

BEM Bayesian Estimation Methods

CMLE Conditional Maximum Likelihood Estimation

CTT Classical Test Theory

DIF Differential Item Functioning

EAP Expected A Posteriori

GPCM Generalized Partial Credit Model

GRM Graded Response Model

ICC Item Characteristics Curve

IRF Item response Function

IRT Item Response Theory

JMLE Joint Maximum Likelihood Estimation

LID Local Item Dependence

MAP Maximum A Posteriori

MCM Multiple Choice Model

MLE Maximum Likelihood Estimation

MMLE Marginal Maximum Likelihood Estimation

NCM Nominal Categories Model



PCM Partial Credit Model

RSM Rating Scale Model

SEE Standard Error of Estimate

SSCE Senior School Certificate Examination



#### **ABSTRACT**

The study examined the difficulty and discrimination indices of polytomous items using dichotomous scoring. It determined the effect of the systematic increase in the response levels of polytomous items on ability estimates in Generalized Partial Credit Model (GPCM) and Graded Response Model (GRM). It further compared the scores of GPCM and GRM on the accuracy of ability estimates and investigated which of the scoring formats was more effective in estimating ability. These were with a view to providing information on the ability of GPCM and GRM in improving the accuracy of scores in polytomous Mathematics achievement test items.

The study adopted the survey research design. The population for the study consisted of students who registered for the Senior School Certificate Examination (SSCE) in Lagos State in 2015. A sample of 1015 students was selected. Two Education Districts (EDs) were selected using purposive sampling technique from the six EDs in the State based on availability of federal schools for an inclusive representation of schools. Three schools were selected from each of the EDs using stratified random sampling technique with school ownership as stratum for selection. One intact SS III class from each of the schools was selected using purposive sampling technique based on students' ability groups. The instrument used in the study was an adapted version of May/June (2006-2014) SSCE General Mathematics Paper 1 titled Mathematics Achievement Test (MAT). Data collected were analysed using ANOVA and Pearson-r. Furthermore, BILOG and IRTPRO were used to generate item parameters of difficulty, discrimination and guessing.

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The results showed the difficulty and discrimination indices of polytomous items using

two different formats. Using dichotomous scoring in the first format difficulty and

discrimination indices were high (-0.73 to 1.95 and 0.38 to 1.72 respectively) while they were

higher in the second format (0.09 to 3.33 and 0.43 to 1.85 respectively). The results also

showed that there was a significant effect of the systematic increase in the response levels of

the items in GPCM on Mathematics ability estimates (F= 3.98, p<0.05) in the first format,

while there was no significant effect in the second format (F=3.16, p>0.05). The results further

showed that there was a significant effect of the systematic increase in the response levels of

the items in GRM on Mathematics ability estimates (F = 22.46, p<0.05 and F = 31.55, p<0.05)

in the first and second formats respectively. Furthermore, GPCM estimated ability more

accurately than GRM (r = 0.22, p<0.05). Finally, the results indicated that the first format

which contained items with one partially correct answer, one full correct answer and two

distracters was more effective in estimating ability (r = 0.22, p<0.05).

The study concluded that Generalized Partial Credit Model had greater ability than

Graded Response Model in improving the accuracy of scores in polytomous Mathematics

achievement test items.

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#### **CHAPTER ONE**

#### INTRODUCTION

#### **Background to the Study**

The inevitability of tests in fulfilling important needs in the decision-making process permeating all facets of human endeavour seems to have made it captured the attention of the public, and at the same time inspires so much anxiety from classroom to work environment. Even the least interested citizen is aware of the growing use of testing in every facet of human endeavour; it is therefore expected that tests attained prominence and usage as far back as 3000 years ago. Educators, governments and establishments' continuous search for reliable and trusted means through which students and applicants could be held accountable of a self-acclaimed ability resulted in testing. Although, several schools of thought have argued that test or examination is not the best measure of a person's ability, while on the other hand, researchers (e.g. Lumsdan, 1978; Kaplan and Saccuzzo, 2005, Afolabi, 2012) have maintained that it is everywhere evident that there is yet no viable alternative to tests in determining a person's academic ability.

A test may be defined as standard set of items which are specific stimuli to which a person overtly responds and which can be scored. There are different types of tests, with different peculiarities, strengths and weaknesses. The major and broad classification of test types is the essay and objective tests. From a practical point of view, the essay (free-response) test provides good measure of students' ability, in so far as it gives them the opportunity to respond freely to items in their own words, thereby building their reasoning and other skills in higher-thinking learning. The required response can be as simple as the writing of a single word



or as complex as the design of a laboratory experiment to test a scientific hypothesis. It helps to recognise and reward different abilities of students through the assigning of partial credits to incomplete understanding of concepts. Despite the enormous credits accrued to the essay tests however, it has been critiqued on the basis of its difficulty to use; its scoring could be subject to raters' bias and inconsistency. On the other hand, the advocates of the objective tests type among which is the multiple-choice format have adduced reasons for its dominant use which includes; limited amount of testing time, its ability to sample a broad range of content and provide a good sample of test takers' knowledge. The responses can be scored by machine, making the scoring process faster and inexpensive, with no room for differences of opinion. In other words, objective tests allow the evaluation of a greater breadth of content in a fixed testing time under limited financial budgets.

Multiple-choice test (MCT), a variant of the objective test has gained credence from classroom assessment to professional licensure examinations (Scott, 2011). It is apt to note that the use to which MCT will be put, would determine the structure of the items in terms of construction of the stem, the response options (correct answer(s)) and the distracters which are predicated on its scoring. For the vast majority of multiple-choice tests, items are scored dichotomously (i.e. correct or incorrect). According to Osterlind and Everson (2009), items with two categories or values (possibilities to respond) are called dichotomous. In this stance, it involves presenting test question and a list of alternatives ranging from three, to four or five as the case may be. The testees are to make free choice of one correct answer from the alternatives given to the item. If an examinee endorse the correct answer, in dichotomous scoring, he/she will be awarded a score of one, whereas if an incorrect answer a score of zero.



Generally, most users of the multiple-choice tests employ dichotomous scoring which are also analysed using the Classical Test Theory (CTT). It is the approach that most users have been exposed to throughout their education. In CTT the respondent's observed score on a whole instrument is the unit of focus. Therefore, a person's score is the unweighted sum of his/her responses to an instrument's items. However, educational reform efforts have led to an increased search of alternatives to the traditional dichotomously scored multiple-choice items, as there has long been a need to assess objectives that require more than a single response (Albanese, 1993). Researchers in a bid to gainfully combine the two item types (i.e essay and objective test), explored the blend of the duo item responses and established that they produced a total score that is more reliable than scores separated by item type. Despite the alternative mixed-item format, they are still subject to both item types' strength and weaknesses. Nonetheless, polytomous tests may perhaps have been observed to provide more equitable approaches to testing than the dichotomously scored multiple-choice tests. They have also been adjudged to provide more information regarding the precision of trait-level estimation than dichotomous items (Jodoin, 2003; Penfield & Bergeron, 2005). The polytomous item type could therefore be perceived as the mediator between the dichotomously scored multiple-choice item and the essay test type as it is contingent on the combined strengths of both item types in a bid to shrink their weaknesses. In polytomous models, items in the test are not just scored right or wrong; instead, each of the categories of responses is evaluated and scored according to its degree of correctness or the amount of information provided toward the full answer. By implication, the items are constructed in such a way that it allows every examinee's efforts at items to be rewarded. Weights are assigned to options in ascending order as examinees knowledge on the item increases, in other words, polytomous items are expected to increase test validity. In a simulation



study, Lau & Wang (1998) observed that classification accuracy in computerized testing situation resulted in higher precision for polytomous items than dichotomous items. They found that lower false negative and false positive classification

For more information, please contact ir-help@oauife.edu.ng