TWO-DIMENSIONAL THEORY FOR A TRANSVERSELY ISOTROPIC THIN PLATE IN NONLINEAR ELASTICITY

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Abstract

This study investigated the effects of finite deformation of a thin plate made of semilinear material of John's type, transversely-isotropic in structure and designed with the aid of a homogenized theory. This was with a view to deriving a two-dimensional theory for the thin plate from a three-dimensional finite elasticity theory.

The hypothesis of hyperelasticity of Cauchy-Truesdell was invoked on the plate's energy stored function. The Frechet's derivative of this function with respect to the energy conjugate geometry of deformation was employed to set up nonlinear three-dimensional boundary-value problem. The three-dimensional boundary-value problem was transformed into an equivalent variational problem and divergence theorem for tensor fields was used to introduce the boundary conditions into threedimensional variational form. The Kirchhoff-Love assumptions were imposed on the space of variations and inner product of tensor fields was applied in the reduction of the three-dimensional variational problem into two-dimensional variational form. The Green's formula was employed to obtain an equivalent two-dimensional boundaryvalue problem for plate in consideration. In the decomposition of gradient of deformation, the Polar decomposition theorem and minimum property of rotation tensor were used to factor the gradient of deformation into product of rotation tensor and stretch symmetric tensor. The flexure and von-Karman conditions were used in the development of two-

dimensional flexural and membrane equations of thin plate respectively. In the finite element formulation, a dual variable was introduced to decompose the flexural equation of plate into two coupled second order equations. For the purpose of numerical computation, a four-node quadrilateral mixed plate element with two-degree of freedom per node was formulated.

The result showed that plate in finite deformation resisted externally applied loads by bending moments, twisting moments, in-plane stresses and transverse shear stresses. Thus, the nonlinear plate developed in-plane and out-of-plane forces and the derived two-dimensional theory reduced to the classical Kirchhoff-Love plate theory. The existence of transverse shear forces in the plates indicated that lines originally normal to the middle surface of the plates before deformation might not remain normal to the deformed surface and the presence of in-plane forces indicated that middle surface of nonlinear plate might experience change in length. The existence and uniqueness of solutions to both the continuous and discrete flexural problems of nonlinear plate in consideration were established. Furthermore, nonlinear plate in flexure exhibited harmonic forces within its planes and the associated membrane problem was described by mid-plane displacements $\zeta_i = 0, i = 1, 2, 3$. Among others, the transversal isotropy of the plate in consideration increased the transverse shear stresses, in-plane forces, axial and flexural rigidities. In the case of degeneracy to isotropy, the obtained flexural rigidity coincided with those of Poisson-Kirchhoff's and Fopp-von Karman's plate theories.

The study concluded that the numerical results demonstrated the efficiency of the Galerkin's finite element approach. It also reinforced the view that finite

deformation approach to elasticity problems provided ample opportunity to reveal important effects which small deformation theory often failed to apprehend.

Keywords: Galerkin's finite element / Fopp-von karman. /Isotropy/Nonlinear Elasticity

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Chapter 1

INTRODUCTION

1.1 Preamble

A thin plate is a structural member having the middle surface in the form of a plane and whose thickness is sufficiently small compared with its other two dimensions (Amenzade (1979); Jawad (1994); Ventsel and Krauthammer (2001)). The loadcarrying action of plates is similar, to certain extent, to that of beams or cables; thus, plates can be approximated by the gridwork of an infinite number of beams or by the network of an infinite number of cables, depending on the flexural rigidity of the structures (Jawad (1994); Ventsel and Krauthammer (2001)). The increasing use of thin plate structures in many branches of technology such as civil, mechanical, aeronautical, marine, and chemical has prompted intensive research in the fields of Engineering, Physics and Applied Mathematics (Wu et al. (1987); Shadnam et al. (2001); Reddy and Arciniega (2004); Tian et al. (2009); and Ayankop (2012)). The widespread applicability of plate structures arises from their intrinsic properties, and based on the fact that when suitably designed, very thin plates can support large loads. Thus, they are utilized in structures such as pressure vessels, aircrafts, bridge decks, missiles, powerplant duct assemblies, submarine bulkheads, building slabs, machine parts, ship and barge hulls, to name but a few (Srinivas (1970); Jawad (1994), and Liew and Kitipornchai (1995)).

The theory of elasticity, in its fundamental formulations, describes the statics and dynamics of three-dimensional elastic bodies. It is well-known that such fundamental models are extremely complex, due to both dimensionality and nonlinearity. These intrinsic complexities have motivated over the years, the development of simplified or reduced theories of elasticity (Reissner (1944, 1945); Reddy (1984); Ciarlet (1997); and Efrati et al. (2009)). In particular, models of lower spatial dimension have been developed to describe the mechanics of slender bodies such as beams, plates and shells (Kant (1982) and Ciarlet (1997)). Furthermore, there are two main reasons why these lower dimensional theories are often preferred to the three-dimensional theory that they approximate (when the thickness or diameter of the cross section is small enough). The one is their simpler tractable mathematical structure, which in turn generates a richer variety of results, while the other is their far better amenability to numerical computations. For instance, to directly approximate the three-dimensional displacement field of a cooling tower seems out of reach at this present time, even in the linear elastic framework (Ciarlet (1997)) till date, the existing codes use two dimensional equations, such as those of W. T. Koiter (Koiter (1970); and Bernadou (1994)). Likewise, substantial progress has recently been achieved for directly approximating three-dimensional displacement field of a linearly elastic rectangular plate, for current codes are almost invariably based on