AN INVESTIGATION OF A THEORY OF STRAIN-GRADIENT PLASTICITY ACCOUNTING FOR ENERGETIC DEPENDENCE ON THE DIVERGENCE OF THE PLASTIC STRAIN

BY

ADEBOWALE SAMUEL BOROKINNI

B.Sc (Mathematics), M.Sc (Mathematics) (OAU)

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ABSTRACT

This work considered strain-gradient plasticity theory. The effects of internal microforces in plastically deformed bodies were investigated through additional explicit dependence of energy function on the divergence of plastic strain.

The principle of virtual power was employed to model the macroscopic and microscopic force balances; first and second law of thermodynamics were used to construct the local dissipation inequality; and von Mises yield criterion, local dissipation inequality together with tensor derivatives of free-energy function were used to establish the constitutive relations. These constitutive relations and microscopic force balance were employed in deriving the governing flow rule for the problem under consideration. For the purpose of numerical solution, a Ritz finite element method was used.

The results obtained showed that internal microforces counterbalance the polar microforces in plastically deformed bodies. Furthermore, the obtained flow rule generalizes the Gurtin-Anand flow rule. In the case of plane strain problem, it was revealed that pure shear can be achieved even if the deformation process is not purely dissipative whereas, pure shear is only achieved for purely dissipative deformation in literature. For a fixed dissipative length scale l, and varying energetic length scales Q and L, the plastic strain in the plastically deformed bodies at any time t decreases. Finite element solutions for one dimensional visco-plastic problems showed that whenever the rate-sensitivity parameter m = 1, the plastic strain for each time attains a threshold, while the plastic strain increases with time. The finite element solution of a purely energetic process in which m = 0, revealed that the plastic strain is approximately zero at some region of the one dimensional visco-plastic solid.

This study concluded that the internal microforces counterbalance the polar microstresses in plastically deformed bodies.

Keyword: Plastic strain/ Strain gradient/ Polar microstresses /Internal microstresses

Supervisor: Prof. A P Akinola

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Chapter 1

INTRODUCTION

1.1 Preamble

Plasticity is the study of stress and strain distributions in plastically deformed bodies. A body is plastically deformed if it does not regain its original shape after the removal of loads that cause the body to deform (i.e. materials cannot fully recover their original configuration). The theory of elasticity has been presented within the frame work that the load causing deformation is bounded or restricted to a limit called the elastic limit so that at the removal of loads, the energy stored in the material during deformation is completely released. However, in plasticity, the load causing deformation is allowed to go beyond the elastic limit, entering into the plastic regime. In this regime, the elastic deformation (which results in stretching and the rotation of the material) and plastic deformation (resulting to permanent or irreversible distortion of the material) occur. As a consequence of this, the deformation experienced by a body can be decomposed into two parts: elastic and plastic parts. This

decomposition gives reason why the total strain and the free energy can be written as sum of the elastic and plastic parts. However at the removal of loads, the body undergoes elastic unloading. The deformation resulting from plastic behaviour gives rise to an irreversible or dissipative process so that, the material is seen to be permanently distorted. Since the plastic deformation will lead to permanent rearrangement of lattice structures that make up the entire body, information on the mechanical behaviour of materials at the micron scales will prove very useful (Gurtin and Anand, 2005). However, this is too complex to be modelled directly, and that is why phenomenological models are usually employed. These models include: Classical and the strain gradient theories.

At the moment, the classical theory of plasticity can be said to be well understood. The governing equations are expressed as a linear theory with approximation in space and time obtainable to desired degree of accuracy. This theory thus has a firm mathematical basis which makes it easier for computational methods such as the finite element method to be adopted (Han and Reddy, 2013). The classical theory can serve as a basis for which other theories such as the strain gradient plasticity can draw their models in order to reveal useful information unaccounted for by the classical theory.

The plastic flow laws which describe the governing equation for a body undergoing plastic deformation are generally presented in two alternative forms. These are the primal and dual formulations. The primal formulation is a kinematically based formulation whose variables of interest are displacements and strains. On the other hand, the dual formulation has, as its variables of interest, the force-like variable such as stress. Any of the formulations can be used to obtain the flow rule for plastic deformation depending on the variables of interest.

The method adopted in many literatures has been the primal formulation approach (Han *et al*, 1997; Ebobisse *et al*, 2008), likely to account for kinematically based internal variables related to the energy function and, the stress through the constitutive relations. Thus, the flow rule or the governing equation for plastic deformation due to the internal variables can be obtained through