EXISTENCE OF FIXED POINTS FOR A CLASS OF s-CONTRACTIONS IN GENERALIZED METRIC SPACE

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ABSTRACT

This study introduced a new and larger class of contraction mappings and determined conditions for existence and uniqueness of fixed points for the class of contraction mappings introduced. This is with a view to establishing the conditions for the existence and uniqueness of the fixed points for a class of $A\Phi$−contractions in G−metric space and the method of their approximation.

A contractive condition was defined. Some of existing ones in G−metric space was generalized. An iterative sequence was generated by the use of the defined contractions. The required conditions for the sequence to be Cauchy and convergent were obtained, and weak compatibility conditions were used in places where more than one operator were considered to approximate the common fixed points.

The $A\Phi$−contractions was found to be more general than Φ−contractions and larger than the A-contractions. Since generalized metric space was wider than the metric space, the corresponding results obtained extended those of the metric space.

This study concluded that the existence of fixed points for this class of mappings was possible, if the generalized metric space was symmetric. Fixed points existed and unique for each of the contraction mappings.

Keywords: Mapping/Contraception mapping/G- metric space/$A\Phi$-contractions

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CHAPTER ONE

INTRODUCTION

1.1 General Background

The fixed point theory is one of the most powerful tools of modern mathematics. It comprises the study of analysis, topology and geometry. It has been developed through different spaces ranging from topological space, set theory and fuzzy topological space. The presence or absence of fixed point is an intrinsic property of a function. However, many necessary and sufficient conditions for the existence of such points involve a mixture of algebraic and topological properties of the mappings and its domain

A point is often called fixed point when it remains invariant irrespective of the type of transformation it undergoes. Let X be a nonempty set and let T be a self-map of X. Then any point $x \in X$ such that $Tx = x$ is called a fixed point of T in X

Poincare (1886) was the first to work in this popular field. Then followed by Brouwer in 1912 who proved fixed point theorem for the solution of the equation $fx = x$. Fixed point theorems for a square, a sphere and their $n-$dimensional counterparts were also proved by Brouwer. The basic contraction mappings which was considered as fundamental principle in field of functional analysis came into existence through Banach (1922). He proved that the contraction mappings in complete metric space posesses a unique fixed point.

Banach contraction principle has been extensively used to study the existence of solution for nonlinear equations and to prove the convergence of algorithms in Computational Mathematics. These applications make fixed point theory significant. Therefore, some mathematicians have been propelled to contribute enormously in the field through generalization, modification and extension of the basic contraction mappings, finding the fixed point(s) of self-mappings and nonself-mappings defined on several ambient spaces, satisfying a variety of conditions. As a result, several fixed point theorems were developed. Some of these theorems provide a constructive method for finding fixed points. They also provide information on the convergence rate along with error estimates. Commonly, the iterative procedures serve as constructive methods in fixed point theory. It is furthermore, of crucial importance to have both a priori and a posteriori estimates of such methods.

Fixed point theory is an interdisciplinary topic which can be applied in various disciplines of mathematics and mathematical sciences such as economics, optimization theory and approximation theory.

With the discovery of computer and development of new softwares for fast computing, a new dimension has been given to the fixed point theory. It has become the subject of scientific research both in determination of economic decision as well as fuzzy and stochastic circumstances. The introduction of Jungck's fixed point theorem on commutative mappings, its relaxation to weak commutativity by Sessa together with the coming up of some latest iterative processes (such as Jungck-Ishikawa iteration, S-T iteration etc) which makes the convergence of fixed point approximation process easily achievable, a new turn has taken place in the field of study up till today.

1.2 General Types of Mappings in Fixed Point Theory

1.2.1 Contraction Mapping Principle

Let (X, d) be a metric space. A map $T: X \to X$ is a contraction mapping or a contraction if there exists a constant $c \in [0, 1)$, such that

$$d(Tx, Ty) \leq cd(x, y) \qquad (1.1)$$

for all $x, y \in X$