A DYNAMIC PROGRAMMING ANALYSIS OF A MODIFIED NANO BACKGAMMON BOARD GAME

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Abstract

This study identified the salient problems involved in the analysis of the full backgammon game especially with respect to the nano backgammon and in this connection constructed an appropriate distance measure of closeness to winning, generated associated recurrence equation from state to state and devised an optimal strategy to maximize the probability of winning over finite number of stages for the players. This was with a view to providing a unified analysis of a unique version of the nano backgammon game.

Having identified the problem as amenable to dynamic programming methodology, two distance measure criteria namely Euclidean and a particular Minkowski with $p = 0.5$ were adopted by two players (player I and player II) in determining the rational movement of the checkers on the board as driven by throws of a fair die. For illustrative purpose, simulation runs of the modified nano backgammon game played by different starting players, at different starting states (167257 and 027127) using different strategies were presented. The problem of maximization was considered using a generated recurrence equation to obtain the optimal solution via the dynamic programming technique. The total duration of time to win, number of wins and proportion of wins by the two players among other factors, were noted. Two hypotheses were formulated and the proportion of wins by player I was tested at 0.05 level of significance using the t-test statistic. As an alternative, a hypothesis was formulated and the relative dependence of duration of time to win on starting states, starting players, winning players and criteria were tested at 0.01 significant level using the χ^2 test statistic.

For the different starting states 167257 and 027127, the total duration of time before conclusion of the game depended on who played first using the Euclidean distance measure criterion. In contrast, while with the starting state 167257, the total duration of time before conclusion of the game appeared not to depend on who started the game if the particular Minkowski distance measure criterion was applied. This was not the case with the starting state 027127. The study also noted that, with respect to the starting states considered, the proportion of wins by player I did not depend on who played first using the particular Minkowski distance measure criterion but with Euclidean distance measure criterion. It depended on who played first. In addition, the duration of time to win depended on starting state, starting player and winning player at 0.01 significant level.

The study concluded that there was significant association of the duration of time to first win and chance of winning with starting state, starting player and winning player for a specific distance measure criterion.

Keywords: . Modified nano/ Backgammon/ Board game/ Minkowski

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Chapter 1

INTRODUCTION

This thesis is concerned with the analysis of strategies involved in the process of playing a modified version of nano backgammon board game. Nano backgammon game itself, is a simplified version of backgammon– one of our oldest games. The specific technique applied here is sequential and has to do with Dynamic Programming or Multistage Programming. We begin by discussing dynamic programming, game theory as well as the backgammon game.

1.1 Introduction to Dynamic Programming

The term "dynamic programming", according to Denardo (2003), was originally used in the 1940s by Richard Bellman to describe the process of solving problems where one needs to find the best decisions one after another. By 1953, he refined this to the modern meaning, referring specifically to nesting smaller decision problems inside larger decisions, and the field was thereafter recognized by the Institute of Electrical and Electronic Engineering (IEEE), as a system analysis and engineering topic. According to Powell (2012), Richard Bellman in 1957 published his seminal volume that laid out a simple and elegant model and algorithmic strategy for solving sequential stochastic optimization problems. This problem can be stated as one of finding a policy $\pi : s \rightarrow A$ that maps a discrete state $s \in S$ to an action $a \in A$, generating a contribution $C(s, a)$. The system then evolves to a new state s^0 with probability $p(s^0|s, a)$. If V(s) is the value of being in state s, then Bellman showed that

$$V(s) = \max_{a \in A}(C(s,a) + \gamma \sum_{s'} P(s'|s,a)V(s'))$$

where γ is a discount factor. This is known as the Bellman's equation (also called the dynamic programming equation) named after its discoverer, Richard Bellman. It is a central result of dynamic programming which restates an optimization problem in recursive form.

1.2 Dynamic Programming.

Often, decision-making process involves several decisions to be taken at different times. For instance, problems of inventory control, evaluation of investment opportunities, longterm corporate planning, and so on, require sequential decision-making. The mathematical technique of optimizing such a sequence of interrelated decisions over a period of time is called dynamic programming.

An example of a dynamic programming problem is given as follows, (Chinneck, 2010):

"An enterprising young statistician believes that she has developed a system for winning a popular Las Vegas game. Her colleagues do not believe that her system works, so they have made a large bet with her that if she starts with three chips, she will not have at least five chips after three plays of the game. Each play of the game involves betting any desired number of available chips and then either winning or losing this number of chips. This statistician believes that her system will give her a probability $\frac{2}{3}$ of winning a given play of the game. The decision at each play should take into account the results of earlier plays and the objective is to maximize the probability of winning her bet with her colleagues".

A dynamic programming solution can be formulated for this problem to determine her optimal policy regarding how many chips to bet (if any) at each of the three plays of the game.

Dynamic programming can be defined as a mathematical procedure designed primarily to improve the computational efficiency of certain mathematical programming problems by decomposing them into smaller, and hence computationally simpler, subproblems. It is a very powerful algorithm paradigm in which a problem is solved by identifying a collection of